

24.2.2

Towards reducing CSAT to 3SAT

Converting $z = x \wedge y$ to 3SAT

z	x	y					
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

Converting $z = x \wedge y$ to 3SAT

z	x	y	$z = x \wedge y$				
0	0	0	1				
0	0	1	1				
0	1	0	1				
0	1	1	0				
1	0	0	0				
1	0	1	0				
1	1	0	0				
1	1	1	1				

Converting $z = x \wedge y$ to 3SAT

z	x	y	$z = x \wedge y$				
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	1	1	1	1
0	1	1	0	0	1	1	1
1	0	0	0	1	0	1	1
1	0	1	0	1	1	0	1
1	1	0	0	1	1	1	0
1	1	1	1	1	1	1	1

Converting $z = x \wedge y$ to 3SAT

z	x	y	$z = x \wedge y$	$z \vee \bar{x} \vee \bar{y}$			
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	1	1	1	1
0	1	1	0	0	1	1	1
1	0	0	0	1	0	1	1
1	0	1	0	1	1	0	1
1	1	0	0	1	1	1	0
1	1	1	1	1	1	1	1

Converting $z = x \wedge y$ to 3SAT

z	x	y	$z = x \wedge y$	$z \vee \bar{x} \vee \bar{y}$	$\bar{z} \vee x \vee y$		
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	1	1	1	1
0	1	1	0	0	1	1	1
1	0	0	0	1	0	1	1
1	0	1	0	1	1	0	1
1	1	0	0	1	1	1	0
1	1	1	1	1	1	1	1

Converting $z = x \wedge y$ to 3SAT

z	x	y	$z = x \wedge y$	$z \vee \bar{x} \vee \bar{y}$	$\bar{z} \vee x \vee y$	$\bar{z} \vee x \vee \bar{y}$	
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	1	1	1	1
0	1	1	0	0	1	1	1
1	0	0	0	1	0	1	1
1	0	1	0	1	1	0	1
1	1	0	0	1	1	1	0
1	1	1	1	1	1	1	1

Converting $z = x \wedge y$ to 3SAT

z	x	y	$z = x \wedge y$	$z \vee \bar{x} \vee \bar{y}$	$\bar{z} \vee x \vee y$	$\bar{z} \vee x \vee \bar{y}$	$\bar{z} \vee \bar{x} \vee y$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	1	1	1	1
0	1	1	0	0	1	1	1
1	0	0	0	1	0	1	1
1	0	1	0	1	1	0	1
1	1	0	0	1	1	1	0
1	1	1	1	1	1	1	1

Converting $z = x \wedge y$ to 3SAT

z	x	y	$z = x \wedge y$	$z \vee \bar{x} \vee \bar{y}$	$\bar{z} \vee x \vee y$	$\bar{z} \vee x \vee \bar{y}$	$\bar{z} \vee \bar{x} \vee y$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	1	1	1	1
0	1	1	0	0	1	1	1
1	0	0	0	1	0	1	1
1	0	1	0	1	1	0	1
1	1	0	0	1	1	1	0
1	1	1	1	1	1	1	1

Converting $z = x \wedge y$ to 3SAT

z	x	y	$z = x \wedge y$	$z \vee \bar{x} \vee \bar{y}$	$\bar{z} \vee x \vee y$	$\bar{z} \vee x \vee \bar{y}$	$\bar{z} \vee \bar{x} \vee y$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	1	1	1	1
0	1	1	0	0	1	1	1
1	0	0	0	1	0	1	1
1	0	1	0	1	1	0	1
1	1	0	0	1	1	1	0
1	1	1	1	1	1	1	1

$$(z = x \wedge y)$$

$$\equiv$$

$$(z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x \vee y) \wedge (\bar{z} \vee x \vee \bar{y}) \wedge (\bar{z} \vee \bar{x} \vee y)$$

Converting $z = x \wedge y$ to 3SAT

z	x	y			
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

Converting $z = x \wedge y$ to 3SAT

z	x	y	$z = x \wedge y$		
0	0	0	1		
0	0	1	1		
0	1	0	1		
0	1	1	0		
1	0	0	0		
1	0	1	0		
1	1	0	0		
1	1	1	1		

Converting $z = x \wedge y$ to 3SAT

z	x	y	$z = x \wedge y$	clauses
0	0	0	1	
0	0	1	1	
0	1	0	1	
0	1	1	0	
1	0	0	0	
1	0	1	0	
1	1	0	0	
1	1	1	1	

Converting $z = x \wedge y$ to 3SAT

z	x	y	$z = x \wedge y$	clauses
0	0	0	1	
0	0	1	1	
0	1	0	1	
0	1	1	0	$z \vee \bar{x} \vee \bar{y}$
1	0	0	0	$\bar{z} \vee x \vee y$
1	0	1	0	$\bar{z} \vee x \vee y$
1	1	0	0	$\bar{z} \vee x \vee y$
1	1	1	1	

Converting $z = x \wedge y$ to 3SAT

z	x	y	$z = x \wedge y$	clauses
0	0	0	1	
0	0	1	1	
0	1	0	1	
0	1	1	0	$z \vee \bar{x} \vee \bar{y}$
1	0	0	0	$\bar{z} \vee x \vee y$
1	0	1	0	$\bar{z} \vee x \vee y$
1	1	0	0	$\bar{z} \vee x \vee y$
1	1	1	1	

$$(z = x \wedge y)$$

\equiv

$$(z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x \vee y) \wedge (\bar{z} \vee x \vee \bar{y}) \wedge (\bar{z} \vee \bar{x} \vee y)$$

Converting $z = x \wedge y$ to 3SAT

Simplify further if you want to

1. Using that $(x \vee y) \wedge (x \vee \bar{y}) = x$, we have that:

$$1.1 \quad (\bar{z} \vee x \vee u) \wedge (\bar{z} \vee x \vee \bar{y}) = (\bar{z} \vee x)$$

$$1.2 \quad (\bar{z} \vee x \vee y) \wedge (\bar{z} \vee \bar{x} \vee y) = (\bar{z} \vee y)$$

2. Using the above two observations, we have that our formula

$$\psi \equiv (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x \vee y) \wedge (\bar{z} \vee x \vee \bar{y}) \wedge (\bar{z} \vee \bar{x} \vee y)$$

$$\text{is equivalent to } \psi \equiv (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x) \wedge (\bar{z} \vee y)$$

Lemma 24.5.

$$(z = x \wedge y) \quad \equiv \quad (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x) \wedge (\bar{z} \vee y)$$

Converting $z = x \wedge y$ to 3SAT

Simplify further if you want to

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$$\text{is equivalent to } \psi \equiv (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x) \wedge (\bar{z} \vee y)$$

Lemma 24.5.

$$(z = x \wedge y) \quad \equiv \quad (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x) \wedge (\bar{z} \vee y)$$

Converting $z = x \wedge y$ to 3SAT

Simplify further if you want to

1. Using that $(x \vee y) \wedge (x \vee \bar{y}) = x$, we have that:

$$1.1 \ (\bar{z} \vee x \vee u) \wedge (\bar{z} \vee x \vee \bar{y}) = (\bar{z} \vee x)$$

$$1.2 \ (\bar{z} \vee x \vee y) \wedge (\bar{z} \vee \bar{x} \vee y) = (\bar{z} \vee y)$$

2. Using the above two observations, we have that our formula

$$\psi \equiv (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x \vee y) \wedge (\bar{z} \vee x \vee \bar{y}) \wedge (\bar{z} \vee \bar{x} \vee y)$$

is equivalent to $\psi \equiv (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x) \wedge (\bar{z} \vee y)$

Lemma 24.5.

$$(z = x \wedge y) \equiv (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x) \wedge (\bar{z} \vee y)$$

Converting $z = x \wedge y$ to 3SAT

Simplify further if you want to

1. Using that $(x \vee y) \wedge (x \vee \bar{y}) = x$, we have that:

$$1.1 \quad (\bar{z} \vee x \vee u) \wedge (\bar{z} \vee x \vee \bar{y}) = (\bar{z} \vee x)$$

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$$\text{is equivalent to } \psi \equiv (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x) \wedge (\bar{z} \vee y)$$

Lemma 24.5.

$$(z = x \wedge y) \quad \equiv \quad (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x) \wedge (\bar{z} \vee y)$$

Converting $z = x \wedge y$ to 3SAT

Simplify further if you want to

1. Using that $(x \vee y) \wedge (x \vee \bar{y}) = x$, we have that:

$$1.1 \quad (\bar{z} \vee x \vee u) \wedge (\bar{z} \vee x \vee \bar{y}) = (\bar{z} \vee x)$$

$$1.2 \quad (\bar{z} \vee x \vee y) \wedge (\bar{z} \vee \bar{x} \vee y) = (\bar{z} \vee y)$$

2. Using the above two observations, we have that our formula

$$\psi \equiv (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x \vee y) \wedge (\bar{z} \vee x \vee \bar{y}) \wedge (\bar{z} \vee \bar{x} \vee y)$$

$$\text{is equivalent to } \psi \equiv (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x) \wedge (\bar{z} \vee y)$$

Lemma 24.5.

$$(z = x \wedge y) \quad \equiv \quad (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x) \wedge (\bar{z} \vee y)$$

Converting $z = x \vee y$ to 3SAT

z	x	y			
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

Converting $z = x \vee y$ to 3SAT

z	x	y	$z = x \vee y$		
0	0	0	1		
0	0	1	0		
0	1	0	0		
0	1	1	0		
1	0	0	0		
1	0	1	1		
1	1	0	1		
1	1	1	1		

Converting $z = x \vee y$ to 3SAT

z	x	y	$z = x \vee y$	clauses
0	0	0	1	
0	0	1	0	
0	1	0	0	
0	1	1	0	
1	0	0	0	
1	0	1	1	
1	1	0	1	
1	1	1	1	

Converting $z = x \vee y$ to 3SAT

z	x	y	$z = x \vee y$	clauses
0	0	0	1	
0	0	1	0	$z \vee x \vee \bar{y}$
0	1	0	0	$z \vee \bar{x} \vee y$
0	1	1	0	$z \vee \bar{x} \vee \bar{y}$
1	0	0	0	$\bar{z} \vee x \vee y$
1	0	1	1	
1	1	0	1	
1	1	1	1	

Converting $z = x \vee y$ to 3SAT

z	x	y	$z = x \vee y$	clauses
0	0	0	1	
0	0	1	0	$z \vee x \vee \bar{y}$
0	1	0	0	$z \vee \bar{x} \vee y$
0	1	1	0	$z \vee \bar{x} \vee \bar{y}$
1	0	0	0	$\bar{z} \vee x \vee y$
1	0	1	1	
1	1	0	1	
1	1	1	1	

$$(z = x \vee y)$$

$$\equiv$$

$$(z \vee x \vee \bar{y}) \wedge (z \vee \bar{x} \vee y) \wedge (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x \vee y)$$

Converting $z = x \vee y$ to 3SAT

Simplify further if you want to

$$(z = x \vee y) \equiv (z \vee x \vee \bar{y}) \wedge (z \vee \bar{x} \vee y) \wedge (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x \vee y)$$

1. Using that $(x \vee y) \wedge (x \vee \bar{y}) = x$, we have that:

$$1.1 (z \vee x \vee \bar{y}) \wedge (z \vee \bar{x} \vee \bar{y}) = z \vee \bar{y}.$$

$$1.2 (z \vee \bar{x} \vee y) \wedge (z \vee \bar{x} \vee \bar{y}) = z \vee \bar{x}$$

2. Using the above two observations, we have the following.

Lemma 24.6.

The formula $z = x \vee y$ is equivalent to the CNF formula

$$(z = x \vee y) \equiv (z \vee \bar{y}) \wedge (z \vee \bar{x}) \wedge (\bar{z} \vee x \vee y)$$

Converting $z = x \vee y$ to 3SAT

Simplify further if you want to

$$(z = x \vee y) \equiv (z \vee x \vee \bar{y}) \wedge (z \vee \bar{x} \vee y) \wedge (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x \vee y)$$

1. Using that $(x \vee y) \wedge (x \vee \bar{y}) = x$, we have that:

$$1.1 (z \vee x \vee \bar{y}) \wedge (z \vee \bar{x} \vee \bar{y}) = z \vee \bar{y}.$$

$$1.2 (z \vee \bar{x} \vee y) \wedge (z \vee \bar{x} \vee \bar{y}) = z \vee \bar{x}$$

2. Using the above two observations, we have the following.

Lemma 24.6.

The formula $z = x \vee y$ is equivalent to the CNF formula

$$(z = x \vee y) \equiv (z \vee \bar{y}) \wedge (z \vee \bar{x}) \wedge (\bar{z} \vee x \vee y)$$

Converting $z = x \vee y$ to 3SAT

Simplify further if you want to

$$(z = x \vee y) \equiv (z \vee x \vee \bar{y}) \wedge (z \vee \bar{x} \vee y) \wedge (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x \vee y)$$

1. Using that $(x \vee y) \wedge (x \vee \bar{y}) = x$, we have that:

$$1.1 (z \vee x \vee \bar{y}) \wedge (z \vee \bar{x} \vee \bar{y}) = z \vee \bar{y}.$$

$$1.2 (z \vee \bar{x} \vee y) \wedge (z \vee \bar{x} \vee \bar{y}) = z \vee \bar{x}$$

2. Using the above two observations, we have the following.

Lemma 24.6.

The formula $z = x \vee y$ is equivalent to the CNF formula

$$(z = x \vee y) \equiv (z \vee \bar{y}) \wedge (z \vee \bar{x}) \wedge (\bar{z} \vee x \vee y)$$

Converting $z = x \vee y$ to 3SAT

Simplify further if you want to

$$(z = x \vee y) \equiv (z \vee x \vee \bar{y}) \wedge (z \vee \bar{x} \vee y) \wedge (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x \vee y)$$

1. Using that $(x \vee y) \wedge (x \vee \bar{y}) = x$, we have that:

$$1.1 (z \vee x \vee \bar{y}) \wedge (z \vee \bar{x} \vee \bar{y}) = z \vee \bar{y}.$$

$$1.2 (z \vee \bar{x} \vee y) \wedge (z \vee \bar{x} \vee \bar{y}) = z \vee \bar{x}$$

2. Using the above two observations, we have the following.

Lemma 24.6.

The formula $z = x \vee y$ is equivalent to the CNF formula

$$(z = x \vee y) \equiv (z \vee \bar{y}) \wedge (z \vee \bar{x}) \wedge (\bar{z} \vee x \vee y)$$

Converting $z = \bar{x}$ to CNF

Lemma 24.7.

$$z = \bar{x} \quad \equiv \quad (z \vee x) \wedge (\bar{z} \vee \bar{x}).$$

Summary of formulas we derived

Lemma 24.8.

The following identities hold:

1. $z = \bar{x} \quad \equiv \quad (z \vee x) \wedge (\bar{z} \vee \bar{x}).$
2. $(z = x \vee y) \quad \equiv \quad (z \vee \bar{y}) \wedge (z \vee \bar{x}) \wedge (\bar{z} \vee x \vee y)$
3. $(z = x \wedge y) \quad \equiv \quad (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x) \wedge (\bar{z} \vee y)$

THE END

...

(for now)