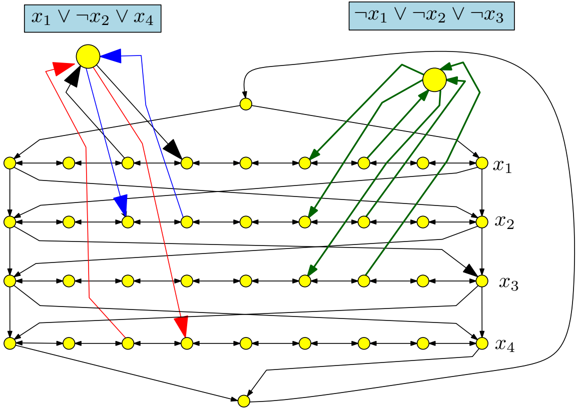
23.3.3

If there is a satisfying assignment, then there is a Hamiltonian cycle

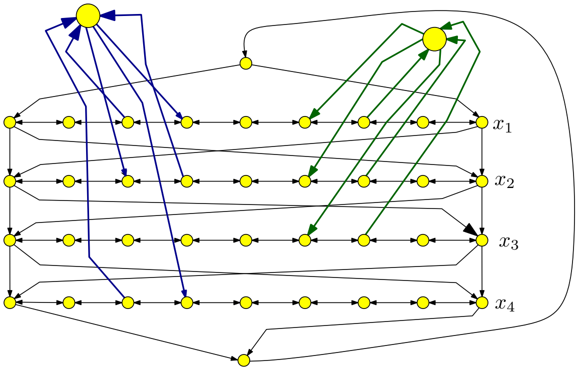
From satisfying assignment to Hamiltonian cycle: By figure



3SAT formula φ:

$$\varphi = \left(x_1 \vee \neg x_2 \vee x_4 \right) \wedge \left(\neg x_1 \vee \neg x_2 \vee \neg x_3 \right)$$

From satisfying assignment to Hamiltonian cycle: By figure



3SAT formula φ:

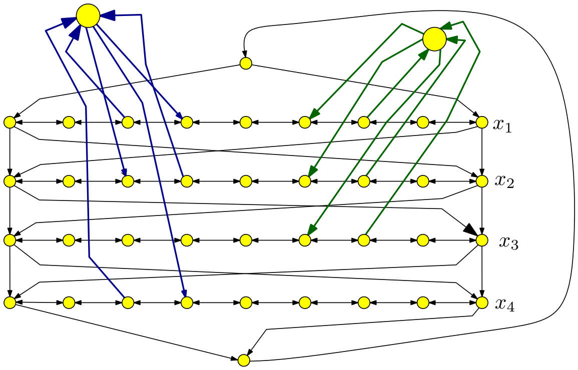
$$\varphi = \left(x_1 \vee \neg x_2 \vee x_4\right) \wedge \left(\neg x_1 \vee \neg x_2 \vee \neg x_3\right)$$

From satisfying assignment to Hamiltonian cycle: By figure

3SAT formula φ:

$$\varphi = \left(x_1 \vee \neg x_2 \vee x_4\right) \wedge \left(\neg x_1 \vee \neg x_2 \vee \neg x_3\right)$$

A satisfying assignment:
$x_1 = \mathbf{0}$, $x_2 = \mathbf{1}$, $x_3 = \mathbf{0}$, $x_4 = \mathbf{1}$

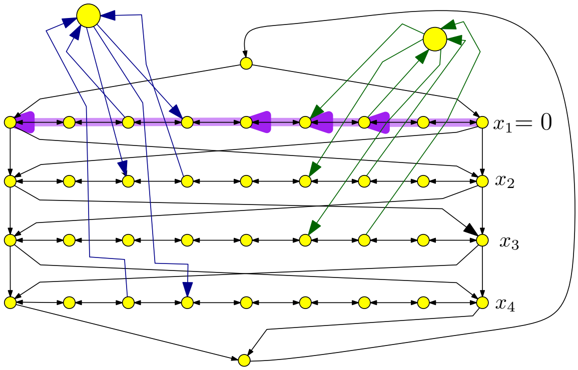


From satisfying assignment to Hamiltonian cycle: By figure

3SAT formula φ:

$$\varphi = \left(x_1 \vee \neg x_2 \vee x_4\right) \wedge \left(\neg x_1 \vee \neg x_2 \vee \neg x_3\right)$$

A satisfying assignment:
$x_1 = 0$, $x_2 = 1$, $x_3 = 0$, $x_4 = 1$

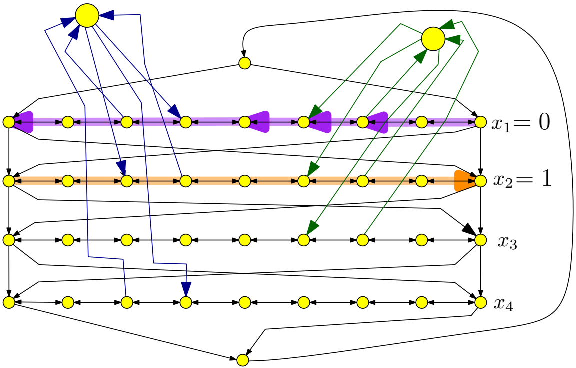


From satisfying assignment to Hamiltonian cycle: By figure

3SAT formula φ:

$$\varphi = \left(x_1 \vee \neg x_2 \vee x_4\right) \wedge \left(\neg x_1 \vee \neg x_2 \vee \neg x_3\right)$$

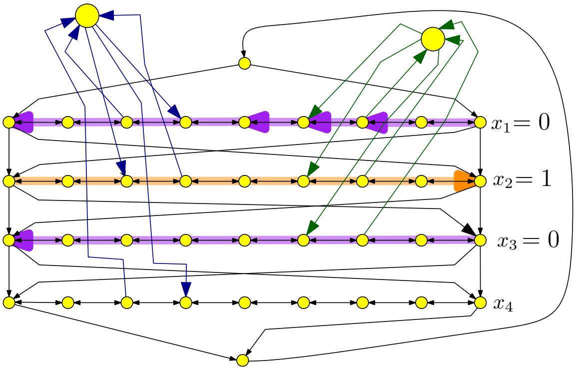
A satisfying assignment:
$x_1 = \mathbf{0}$, $x_2 = \mathbf{1}$, $x_3 = \mathbf{0}$, $x_4 = \mathbf{1}$



From satisfying assignment to Hamiltonian cycle: By figure

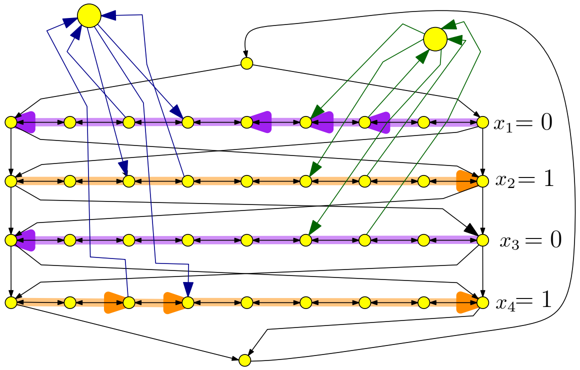
3SAT formula φ:

$$\varphi = \Big(x_1 \lor \neg x_2 \lor x_4\Big) \land \Big(\neg x_1 \lor \neg x_2 \lor \neg x_3\Big)$$



A satisfying assignment:
$x_1 = 0$, $x_2 = 1$, $x_3 = 0$, $x_4 = 1$

From satisfying assignment to Hamiltonian cycle: By figure

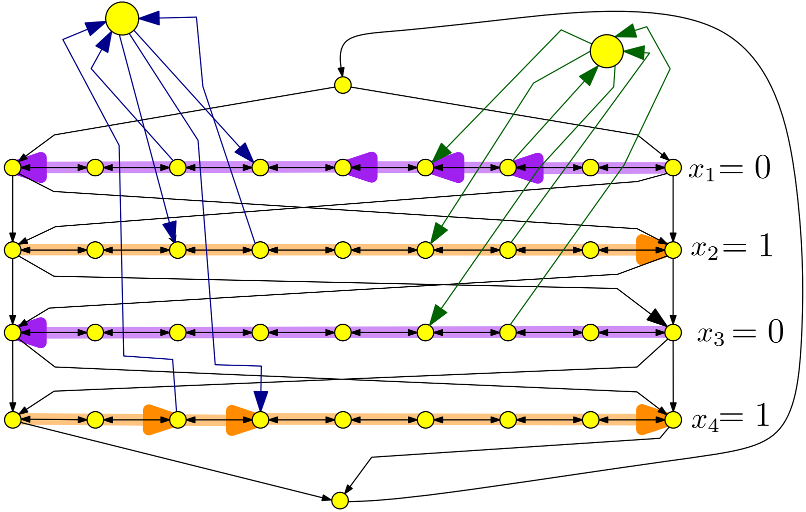


3SAT formula φ:

$$\varphi = \left(x_1 \vee \neg x_2 \vee x_4\right) \wedge \left(\neg x_1 \vee \neg x_2 \vee \neg x_3\right)$$

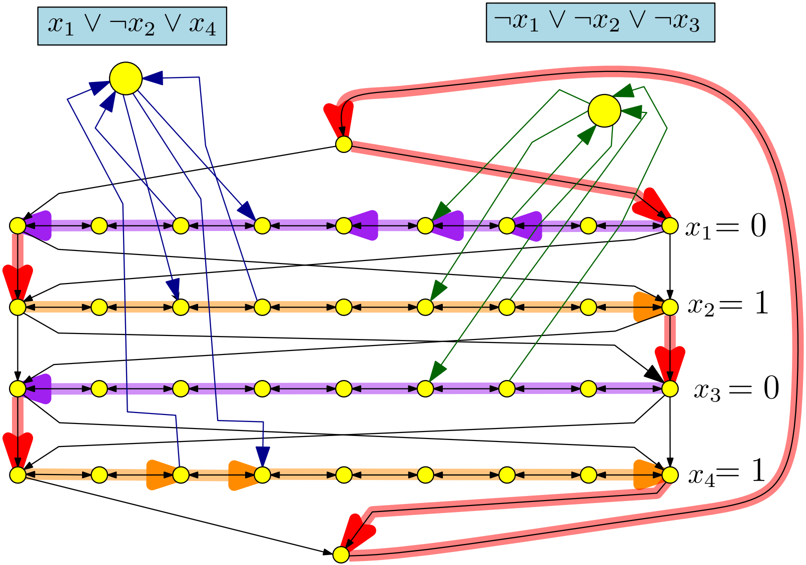
A satisfying assignment:
$x_1 = \mathbf{0}$, $x_2 = \mathbf{1}$, $x_3 = \mathbf{0}$, $x_4 = \mathbf{1}$

Reduction: Satisfying assignment ⇒ Hamiltonian cycle



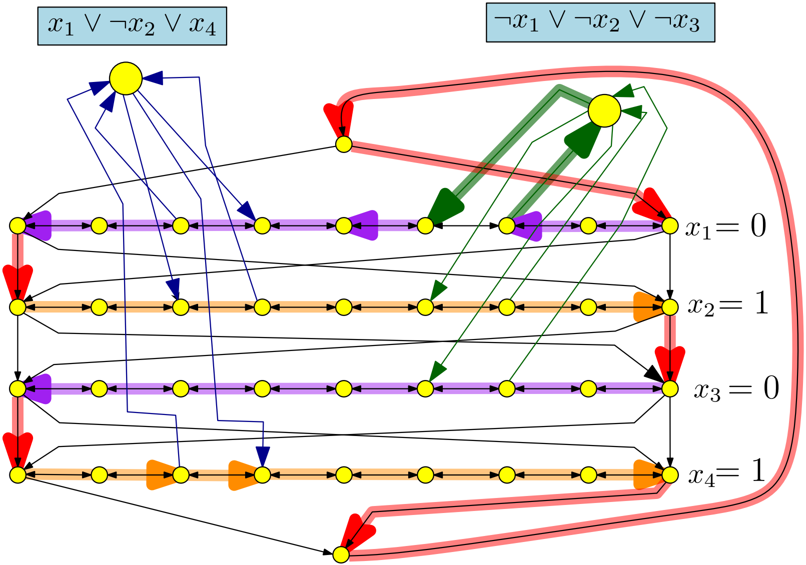
Satisfying assignment: $\mathbf{x_1 = 0}$, $\mathbf{x_2 = 1}$, $\mathbf{x_3 = 0}$, $\mathbf{x_4 = 1}$

Reduction: Satisfying assignment ⇒ Hamiltonian cycle



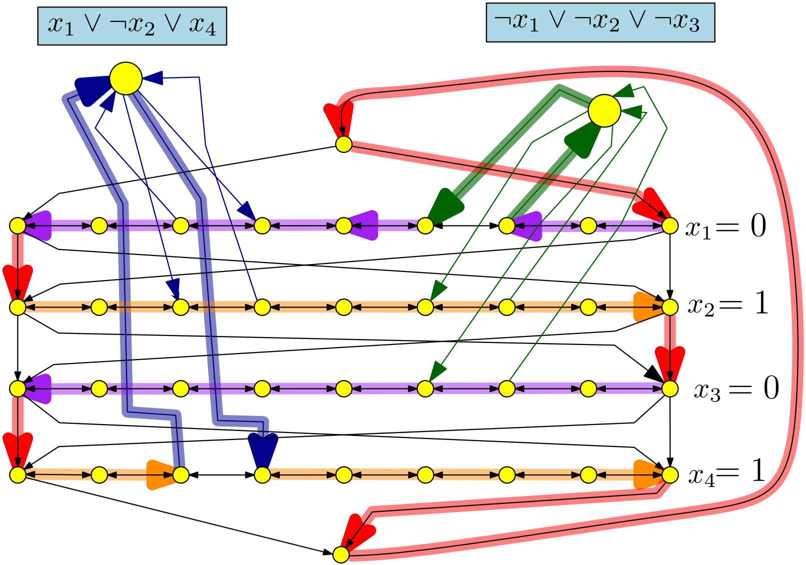
Satisfying assignment: $\mathbf{x_1 = 0}$, $\mathbf{x_2 = 1}$, $\mathbf{x_3 = 0}$, $\mathbf{x_4 = 1}$

Reduction: Satisfying assignment $\Rightarrow$ Hamiltonian cycle



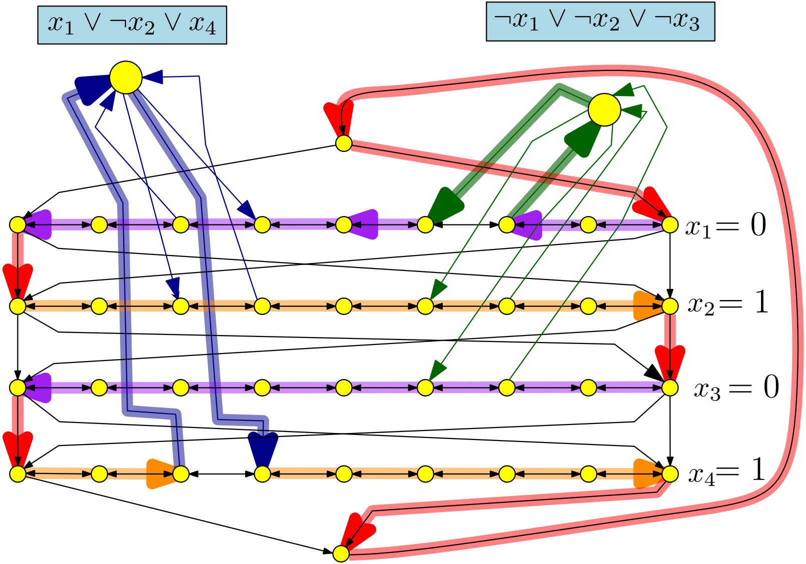
Satisfying assignment: $\boldsymbol{x_1 = 0}$, $\boldsymbol{x_2 = 1}$, $\boldsymbol{x_3 = 0}$, $\boldsymbol{x_4 = 1}$

Reduction: Satisfying assignment ⇒ Hamiltonian cycle



Satisfying assignment: $\boldsymbol{x_1 = 0}$, $\boldsymbol{x_2 = 1}$, $\boldsymbol{x_3 = 0}$, $\boldsymbol{x_4 = 1}$

Reduction: Satisfying assignment $\Rightarrow$ Hamiltonian cycle



Satisfying assignment: $\boldsymbol{x_1 = 0}$, $\boldsymbol{x_2 = 1}$, $\boldsymbol{x_3 = 0}$, $\boldsymbol{x_4 = 1}$

Conclude: If φ has a satisfying assignment then there is an Hamiltonian cycle in G_φ.

Correctness Proof

Lemma 23.1.

φ has a satisfying assignment α $\implies$ G_φ has a Hamiltonian cycle.

Proof.

Let a be the satisfying assignment for φ. Define Hamiltonian cycle as follows

- If $\alpha(x_i) = 1$ then traverse path i from left to right
- If $\alpha(x_i) = 0$ then traverse path i from right to left
- For each clause, path of at least one variable is in the "right" direction to splice in the node corresponding to clause
- Clearly, resulting cycle is Hamiltonian. □

THE END

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(for now)