17.4
Shortest path trees and variants

17.4.1
Shortest Path Tree

Shortest Path Tree

Dijkstra's algorithm finds the shortest path distances from s to $\boldsymbol{V}$.

Question: How do we find the paths themselves?

Q = makePQ()
insert(Q, $(s, 0)$)
$\text{prev}(s) \leftarrow \textit{null}$
for each node $u \neq s$ do
    insert(Q, (u, ∞))
    $\text{prev}(u) \leftarrow$ null

$X = \emptyset$
for $i = 1$ to $|V|$ do
    $(v, \text{dist}(s, v)) = \textit{extractMin}(Q)$
    $X = X \cup \{v\}$
    for each u in $\text{Adj}(v)$ do
        if $(\text{dist}(s, v) + \ell(v, u) < \text{dist}(s, u))$ then
            decreaseKey(Q, $(u, \text{dist}(s, v) + \ell(v, u))$)
            $\text{prev}(u) = v$

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```
Q = makePQ()
insert(Q, (s, 0))
prev(s) ← null
for each node u ≠ s do
    insert(Q, (u, ∞) )
    prev(u) ← null

X = ∅
for i = 1 to |V| do
    (v, dist(s, v)) = extractMin(Q)
    X = X ∪ {v}
    for each u in Adj(v) do
        if (dist(s, v) + ℓ(v, u) < dist(s, u)) then
            decreaseKey(Q, (u, dist(s, v) + ℓ(v, u)))
            prev(u) = v
```

Shortest Path Tree

Lemma

The edge set $(u, \mathbf{prev}(u))$ *is the reverse of a shortest path tree rooted at* s*. For each* u*, the reverse of the path from* u *to* s *in the tree is a shortest path from* s *to* u*.*

Proof Sketch.

1. The edge set $\{(u, \mathbf{prev}(u)) \mid u \in V\}$ induces a directed in-tree rooted at s (Why?)
2. Use induction on $|X|$ to argue that the tree is a shortest path tree for nodes in V.

$\square$

Shortest paths to s

Dijkstra's algorithm gives shortest paths from s to all nodes in V.
How do we find shortest paths from all of V to s?

1. In undirected graphs shortest path from s to u is a shortest path from u to s so there is no need to distinguish.
2. In directed graphs, use Dijkstra's algorithm in G^{rev}!

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THE END

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(for now)