17.3.3
Shortest path in the weighted case using BFS

Single-Source Shortest Paths via BFS

1. **Special case:** All edge lengths are **1**.
   1. Run **BFS**(s) to get shortest path distances from s to all other nodes.
   2. $O(m + n)$ time algorithm.
2. **Special case:** Suppose $\ell(e)$ is an integer for all e?
   Can we use **BFS**? Reduce to unit edge-length problem by placing $\ell(e) - 1$ dummy nodes on e.

Single-Source Shortest Paths via BFS

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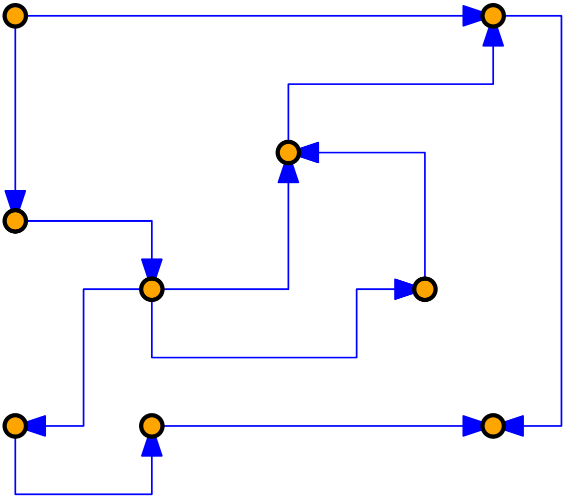
Example of edge refinement



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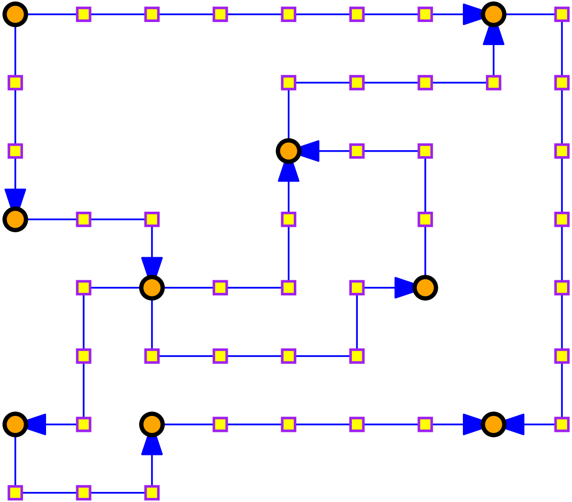

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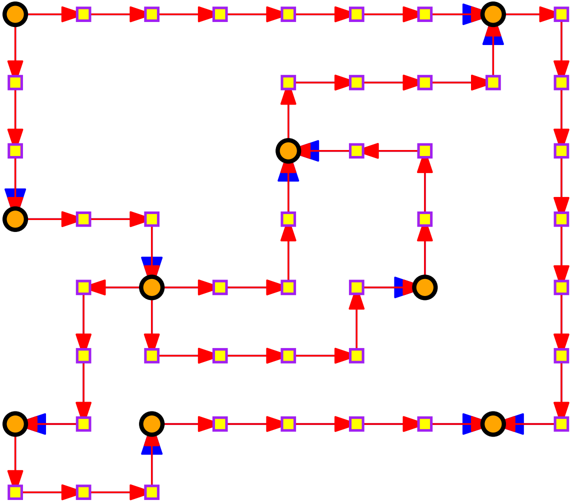

Shortest path using BFS

Let $L = \max_e \ell(e)$. New graph has $O(mL)$ edges and $O(mL + n)$ nodes. **BFS** takes $O(mL + n)$ time. Not efficient if L is large.

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THE END

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(for now)