17.3
Shortest Paths and Dijkstra's Algorithm

17.3.1
Problem definition

Shortest Path Problems

Shortest Path Problems

Input A (undirected or directed) graph $G = (V, E)$ with edge lengths (or costs). For edge $e = (u, v)$, $\ell(e) = \ell(u, v)$ is its length.

1. Given nodes s, t find shortest path from s to t.
2. Given node s find shortest path from s to all other nodes.
3. Find shortest paths for all pairs of nodes.

Many applications!

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Single-Source Shortest Paths:

Non-Negative Edge Lengths

1. Single-Source Shortest Path Problems
   1. Input: A (undirected or directed) graph $G = (V, E)$ with non-negative edge lengths. For edge $e = (u, v)$, $\ell(e) = \ell(u, v)$ is its length.
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   3. Given node s find shortest path from s to all other nodes.
2.
   1. Restrict attention to directed graphs
   2. Undirected graph problem can be reduced to directed graph problem - how?
      1. Given undirected graph G, create a new directed graph G' by replacing each edge $\{u, v\}$ in G by (u, v) and (v, u) in G'.
      2. set $\ell(u, v) = \ell(v, u) = \ell(\{u, v\})$
      3. Exercise: show reduction works. Relies on non-negativity!

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THE END

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(for now)