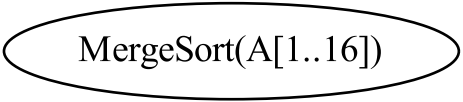
# 10.6.3

# Running time analysis of merge-sort: Recursion tree method

# Recursion tree



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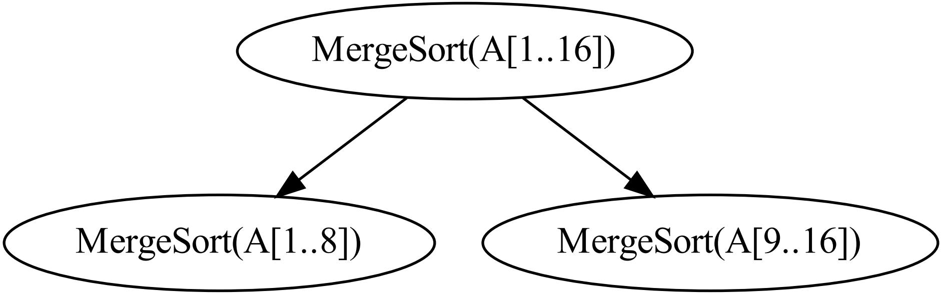

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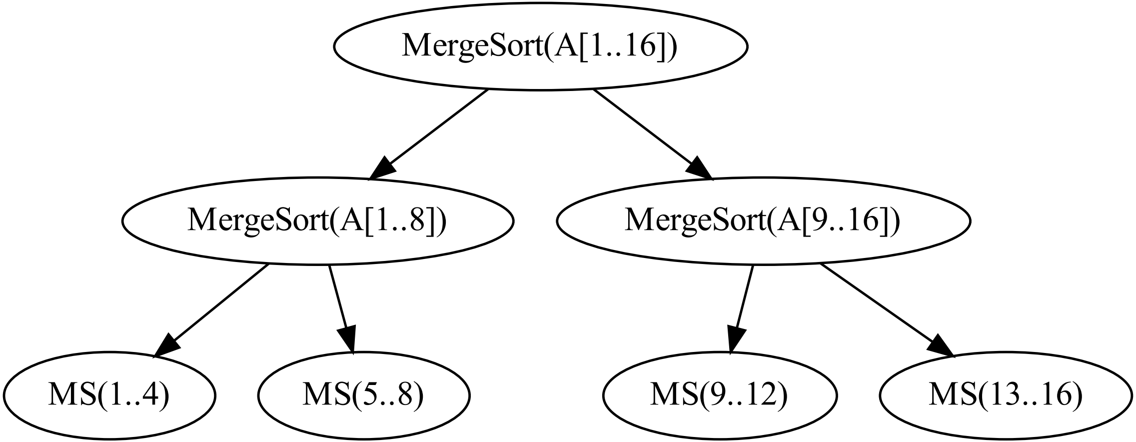

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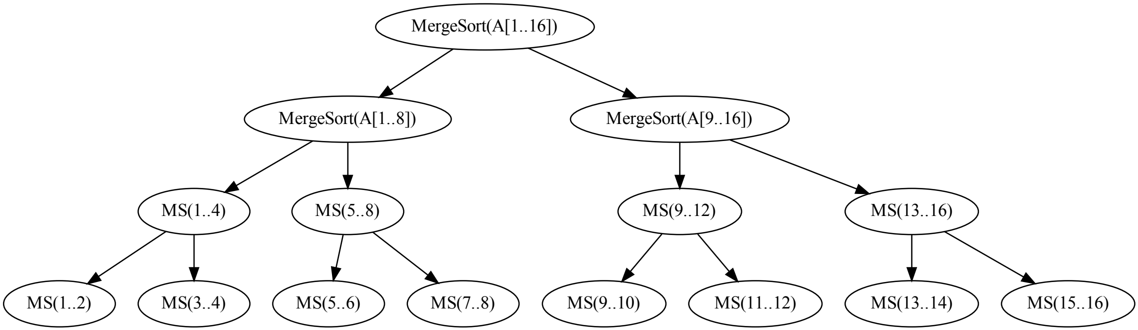

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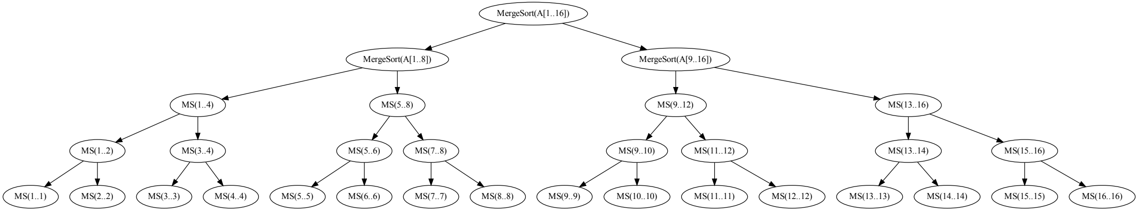
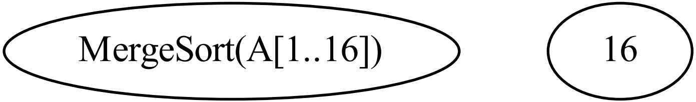


# Recursion tree: subproblem sizes



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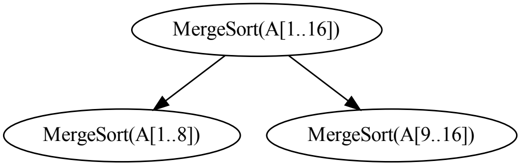


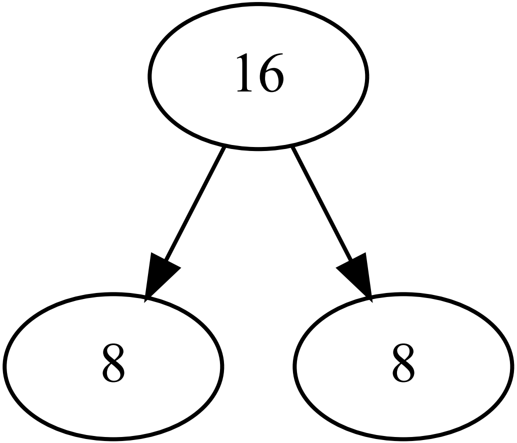

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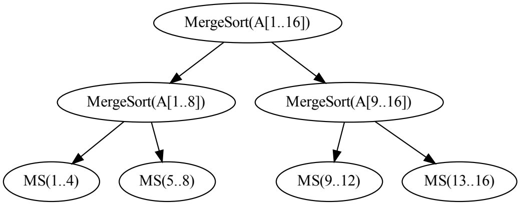


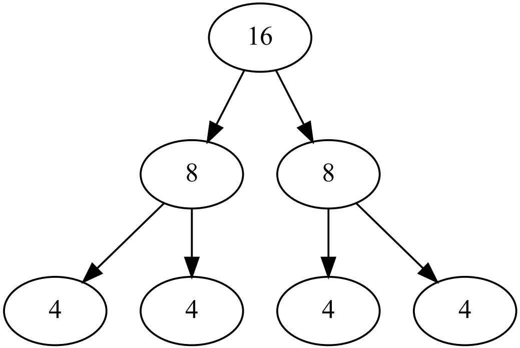

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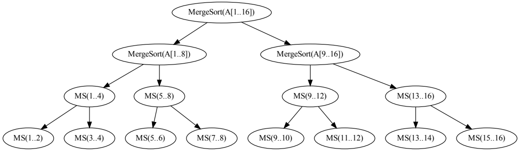


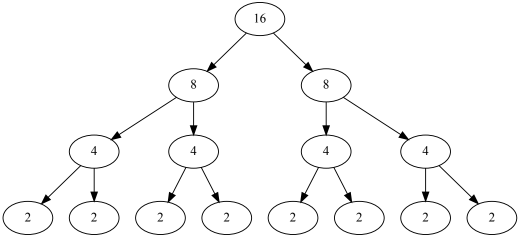

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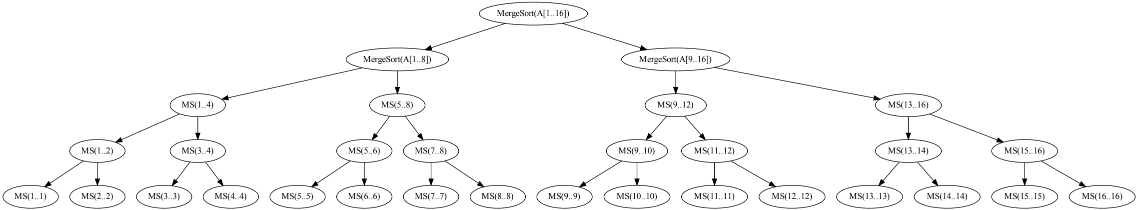
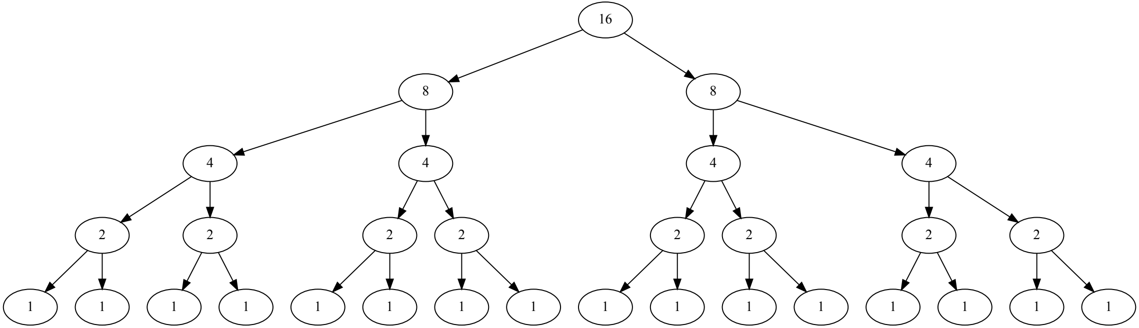
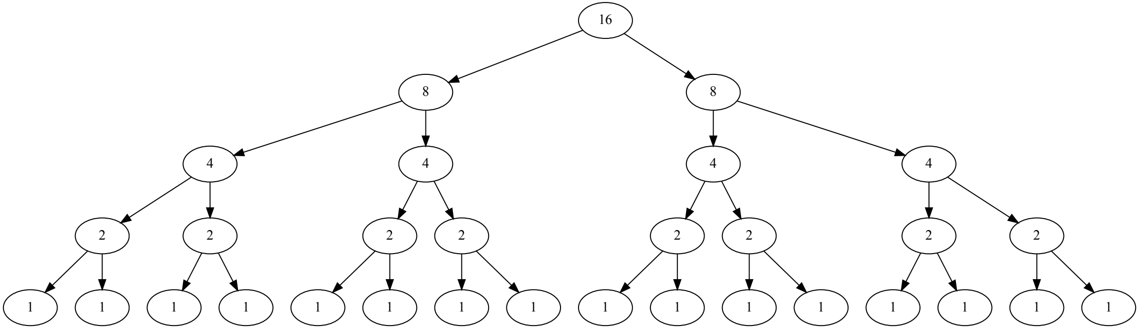


# Recursion tree: Total work?



# Running Time

$T(n)$: time for merge sort to sort an $n$ element array

$$T(n) = T(\lfloor n/2 \rfloor) + T(\lceil n/2 \rceil) + cn$$

What do we want as a solution to the recurrence?

Almost always only an asymptotically tight bound. That is we want to know $f(n)$ such that $T(n) = \Theta(f(n))$.

1. $T(n) = O(f(n))$ - upper bound
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# Solving Recurrences: Some Techniques

1. Know some basic math: geometric series, logarithms, exponentials, elementary calculus
2. Expand the recurrence and spot a pattern and use simple math
3. Recursion tree method — imagine the computation as a tree
4. Guess and verify — useful for proving upper and lower bounds even if not tight bounds

**Albert Einstein:** "Everything should be made as simple as possible, but not simpler."

Know where to be loose in analysis and where to be tight. Comes with practice, practice, practice!

Review notes on recurrence solving.

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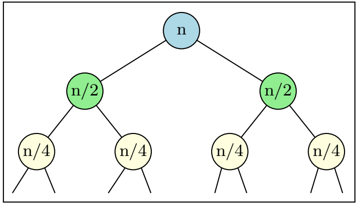
# Recursion Trees

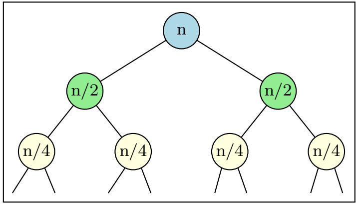
## **MergeSort**: **n** is a power of **2**




1. Unroll the recurrence. $T(n) = 2T(n/2) + cn$

# Recursion Trees

## MergeSort: $n$ is a power of $2$

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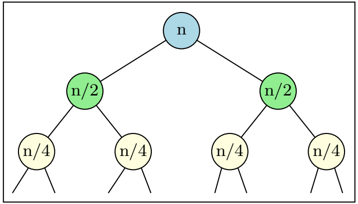
2. Identify a pattern. At the $i$th level total work is $cn$.

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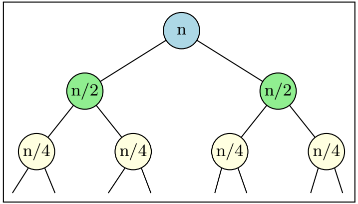


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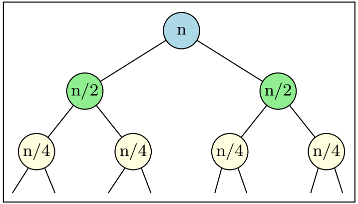


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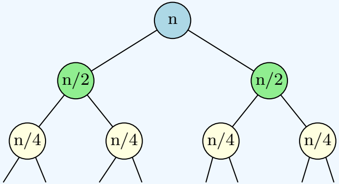
# Recursion Trees

## An illustrated example...



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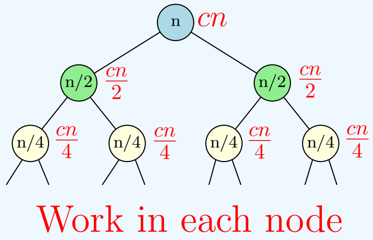

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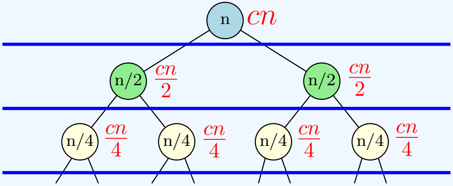


Work in each node

# Recursion Trees

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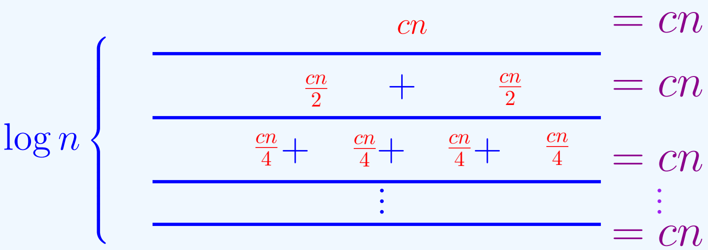

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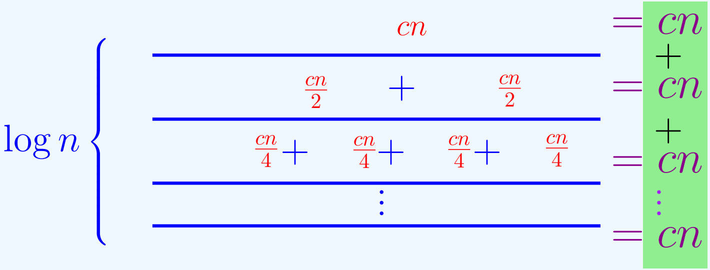


$$= cn \log n = O(n \log n)$$

# Merge Sort Variant

**Question:** Merge Sort splits into 2 (roughly) equal sized arrays. Can we do better by splitting into more than 2 arrays? Say $k$ arrays of size $n/k$ each?

# THE END

...

# (for now)