9.5
Turing complete

Equivalent to a program

Definition

*A system is **Turing complete** if one can simulate a Turing machine using it.*

1. Programming languages (yey!).
2. C++ templates system (boo).
3. John Conway's game of life.
4. Many games (Minesweeper).
5. Post's correspondence problem.

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Post's correspondence problem

S: set of **domino tiles**.

$\dfrac{abb}{bc}$: domino piece a string at the top and a string at the bottom.

Example:

$$S = \left\{ \frac{b}{ca}, \frac{a}{ab}, \frac{ca}{a}, \frac{abc}{c} \right\}.$$

Matching dominos

$$S = \left\{ \frac{b}{ca}, \frac{a}{ab}, \frac{ca}{a}, \frac{abc}{c} \right\}.$$

match for S: ordered list of dominos from S, such that top strings make same string as bottom strings. Example:

$$\frac{a}{ab} \quad \frac{b}{ca} \quad \frac{ca}{a} \quad \frac{a}{ab} \quad \frac{abc}{c}.$$

(1) Can use same domino more than once.

(2) Do not have to use all pieces of S.

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Post's Correspondence Problem

Post's Correspondence Problem (PCP) is deciding whether a set of dominos has a match or not.

modified Post's Correspondence Problem (MPCP): PCP + a special tile.

Matches for MPCP have to start with the special tile.

Theorem

The MPCP problem is undecidable.

THE END

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(for now)