7.4
Some properties of CFLs

7.4.1
Closure properties of CFLs

Bad news: Canonical non-CFL

Theorem

$L = \{a^n b^n c^n \mid n \geq 0\}$ *is not context-free.*

Proof based on pumping lemma for CFLs. See supplemental for the proof.

More bad news: CFL not closed under intersection

Theorem

CFLs *are not closed under intersection.*

Closure Properties of CFLs

$G_1 = (V_1, T, P_1, S_1)$ and $G_2 = (V_2, T, P_2, S_2)$

Assumption: $V_1 \cap V_2 = \emptyset$, that is, non-terminals are not shared

Theorem

CFLs *are closed under union.* L_1, L_2 CFLs *implies* $L_1 \cup L_2$ *is a* CFL.

Theorem

CFLs *are closed under concatenation.* L_1, L_2 CFLs *implies* $L_1 \bullet L_2$ *is a* CFL.

Theorem

CFLs *are closed under Kleene star.*

If L *is a* CFL $\implies$ L^* *is a* CFL.

Closure Properties of CFLs

$G_1 = (V_1, T, P_1, S_1)$ and $G_2 = (V_2, T, P_2, S_2)$
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Closure Properties of CFLs

Union

$G_1 = (V_1, T, P_1, S_1)$ and $G_2 = (V_2, T, P_2, S_2)$
Assumption: $V_1 \cap V_2 = \emptyset$, that is, non-terminals are not shared.

Theorem

CFLs *are closed under union.* L_1, L_2 CFLs *implies* $L_1 \cup L_2$ *is a* CFL.

Closure Properties of CFLs

Concatenation

Theorem

CFLs are closed under concatenation. L_1, L_2 *CFLs implies* $L_1 \bullet L_2$ *is a CFL.*

Closure Properties of CFLs

Stardom (i.e, Kleene star)

Theorem

CFLs are closed under Kleene star.
If L is a CFL $\implies$ L^* is a CFL.

Exercise

- Prove that every regular language is context-free using previous closure properties.
- Prove the set of regular expressions over an alphabet Σ forms a non-regular language which is context-free.

Even more bad news: CFL not closed under complement

Theorem

CFLs *are* *not* *closed under complement.*

Good news: Closure Properties of CFLs continued

Theorem

If L_1 is a CFL and L_2 is regular then $L_1 \cap L_2$ is a CFL.

THE END

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(for now)