6.4

Closure properties: Proving non-regularity

Non-regularity via closure properties

$H = \{\text{bitstrings with equal number of 0s and 1s}\}$

$H' = \{0^k 1^k \mid k \geq 0\}$

Suppose we have already shown that L' is non-regular. Can we show that L is non-regular without using the fooling set argument from scratch?

$H' = H \cap L(0^*1^*)$

Claim: The above and the fact that L' is non-regular implies L is non-regular. Why?

Suppose H is regular. Then since $L(0^*1^*)$ is regular, and regular languages are closed under intersection, H' also would be regular. But we know H' is not regular, a contradiction.

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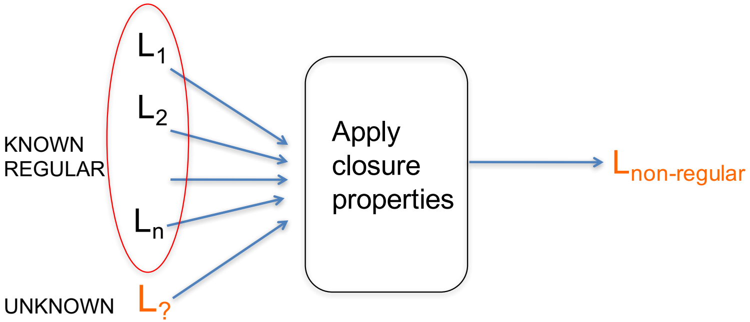
Non-regularity via closure properties

General recipe:



Proving non-regularity: Summary

- Method of distinguishing suffixes. To prove that L is non-regular find an infinite fooling set.
- Closure properties. Use existing non-regular languages and regular languages to prove that some new language is non-regular.
- Pumping lemma. We did not cover it but it is sometimes an easier proof technique to apply, but not as general as the fooling set technique.

THE END

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(for now)