5.1.3
Proof of correctness of conversion of NFA to DFA

Proof of Correctness

Theorem

Let $N = (Q, \Sigma, s, \delta, A)$ *be a* NFA *and let* $D = (Q', \Sigma, \delta', s', A')$ *be a* DFA *constructed from* N *via the subset construction. Then* $L(N) = L(D)$.

Stronger claim:

Lemma

For every string w, $\delta^*_N(s, w) = \delta^*_D(s', w)$.

Proof by induction on $|w|$.

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Proof continued I

Lemma

For every string w, $\delta_N^*(s, w) = \delta_D^*(s', w)$.

Proof:

Base case: $w = \epsilon$.

$\delta_N^*(s, \epsilon) = \epsilon\text{reach}(s)$.

$\delta_D^*(s', \epsilon) = s' = \epsilon\text{reach}(s)$ by definition of s'.

Proof continued II

Lemma

For every string w, $\delta_N^*(s, w) = \delta_D^*(s', w)$.

Inductive step: $w = xa$ (Note: suffix definition of strings)

$\delta_N^*(s, xa) = \cup_{p \in \delta_N^*(s,x)} \delta_N^*(p, a)$ by inductive definition of δ_N^*

$\delta_D^*(s', xa) = \delta_D(\delta_D^*(s, x), a)$ by inductive definition of δ_D^*

By inductive hypothesis: $Y = \delta_N^*(s, x) = \delta_D^*(s, x)$

Thus $\delta_N^*(s, xa) = \cup_{p \in Y} \delta_N^*(p, a) = \delta_D(Y, a)$ by definition of δ_D.

Therefore,

$\delta_N^*(s, xa) = \delta_D(Y, a) = \delta_D(\delta_D^*(s, x), a) = \delta_M^*(s', xa)$. which is what we need.

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□

THE END

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(for now)