NFAs continued, Closure Properties of Regular Languages

Lecture 5

Tuesday, September 8, 2020

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5.1
Equivalence of NFAs and DFAs

Regular Languages, DFAs, NFAs

Theorem

Languages accepted by DFAs, NFAs, and regular expressions are the same.

- DFAs are special cases of NFAs (easy)
- NFAs accept regular expressions (seen)
- DFAs accept languages accepted by NFAs (shortly)
- Regular expressions for languages accepted by DFAs (later in the course)

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Equivalence of NFAs and DFAs

Theorem

For every NFA N there is a DFA M such that $L(M) = L(N)$.

5.1.1
The idea of the conversion of NFA to DFA

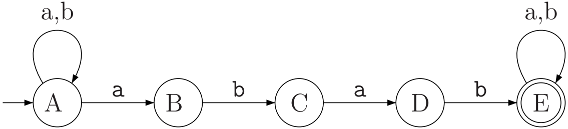
DFAs are memoryless...

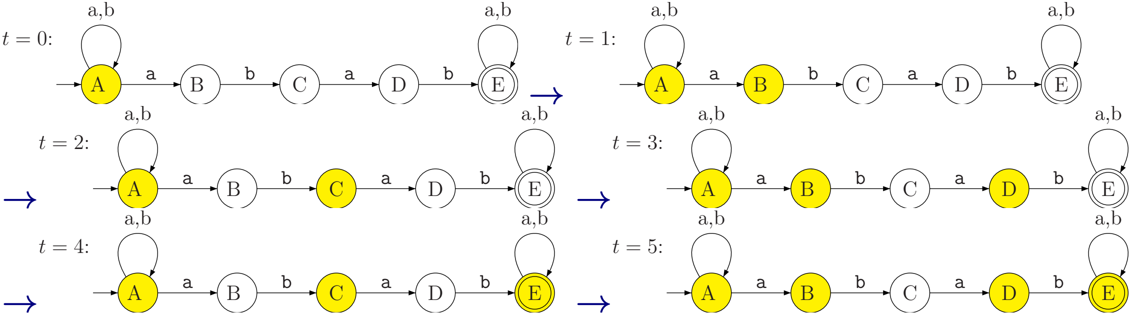
1. DFA knows only its current state.
2. The state is the memory.
3. To design a DFA, answer the question:
   What minimal info needed to solve problem.

Simulating NFA

Example the first revisited

Previous lecture.. Ran NFA[(N1)] on input ***ababa***.

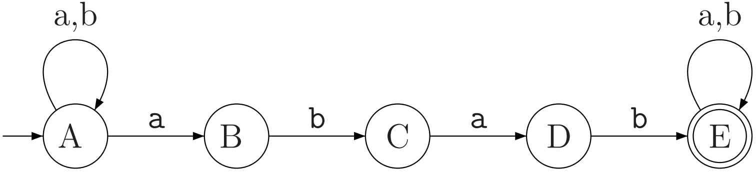




The state of the NFA

It is easy to state that the state of the automata is the states that it might be situated at.

(N1)



configuration: A set of states the automata might be in.

Possible configurations: $\emptyset$, $\{A\}$, $\{A, B\}$...

Big idea: Build a DFA on the configurations.

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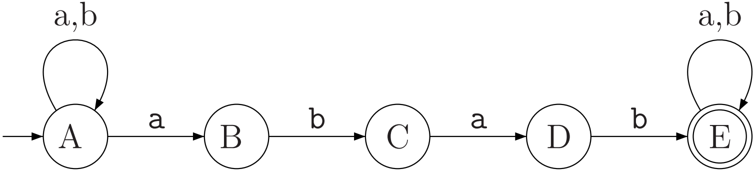

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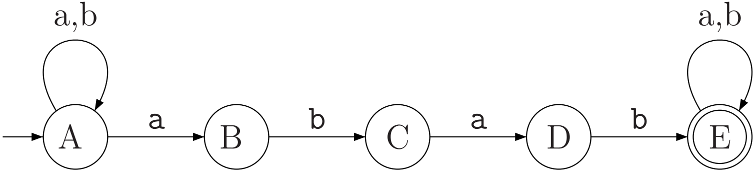

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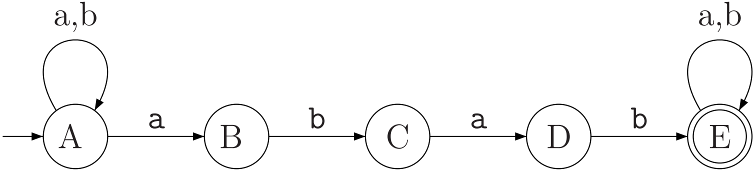

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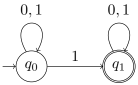
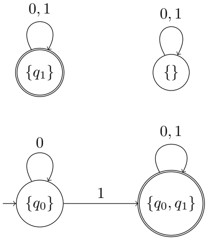
Example





Simulating an NFA by a DFA

- Think of a program with fixed memory that needs to simulate NFA N on input w.
- What does it need to store after seeing a prefix x of w?
- It needs to know at least $\delta^*(s, x)$, the set of states that N could be in after reading x
- Is it sufficient? Yes, if it can compute $\delta^*(s, xa)$ after seeing another symbol a in the input.
- When should the program accept a string w? If $\delta^*(s, w) \cap A \neq \emptyset$.

Key Observation: DFA M simulating N should know current configuration of N.

State space of the DFA is $\mathcal{P}(Q)$.

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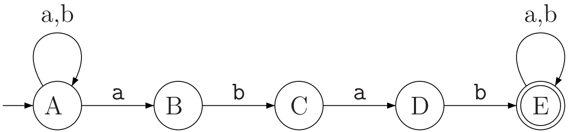
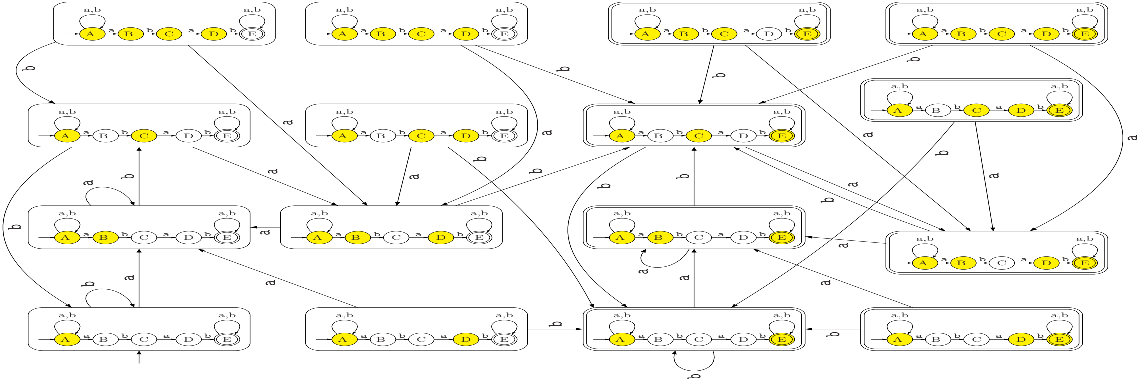
Example: DFA from NFA

NFA: (N1)



DFA:



Formal Tuple Notation for NFA

Definition

A non-deterministic finite automata (NFA) $N = (Q, \Sigma, \delta, s, A)$ is a five tuple where

- Q is a finite set whose elements are called states,
- Σ is a finite set called the input alphabet,
- $\delta : Q \times \Sigma \cup \{\epsilon\} \rightarrow \mathcal{P}(Q)$ is the transition function (here $\mathcal{P}(Q)$ is the power set of Q),
- $s \in Q$ is the start state,
- $A \subseteq Q$ is the set of accepting/final states.

$\delta(q, a)$ for $a \in \Sigma \cup \{\epsilon\}$ is a subset of Q — a set of states.

THE END

...

(for now)