4.1.2
Extending the transition function to strings

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1. NFA $N = (Q, \Sigma, \delta, s, A)$
2. $\delta(q, a)$: set of states that N can go to from q on reading $a \in \Sigma \cup \{\varepsilon\}$.
3. Want transition function $\delta^* : Q \times \Sigma^* \to \mathcal{P}(Q)$
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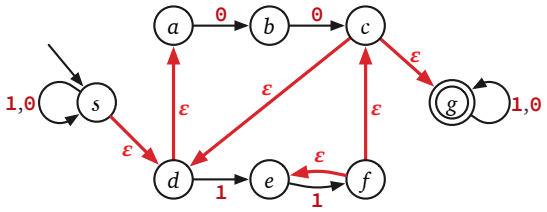
For NFA $N = (Q, \Sigma, \delta, s, A)$ and $q \in Q$ the ϵreach(q) is the set of all states that q can reach using only ε-transitions.



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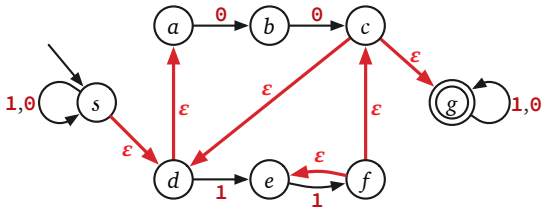

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Definition

Inductive definition of $\delta^* : Q \times \Sigma^* \to \mathcal{P}(Q)$:

- if $w = \varepsilon$, $\delta^*(q, w) = \epsilon\text{reach}(q)$
- if $w = a$ where $a \in \Sigma$: $\delta^*(q, a) = \epsilon\text{reach}\left(\bigcup_{p \in \epsilon\text{reach}(q)} \delta(p, a)\right)$
- if $w = ax$: $\delta^*(q, w) = \epsilon\text{reach}\left(\bigcup_{p \in \epsilon\text{reach}(q)} \left(\bigcup_{r \in \delta^*(p,a)} \delta^*(r, x)\right)\right)$

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Transition for strings: w = ax

Translation...

$$\delta^*(q, w) = \epsilon\text{reach}\left(\bigcup_{p \in \epsilon\text{reach}(q)} \left(\bigcup_{r \in \delta^*(p,a)} \delta^*(r, x)\right)\right)$$

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2. $N = \bigcup_{p \in R} \delta^*(p, a)$: All the states reachable from q with the letter a.
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Formal definition of language accepted by N

Definition

A string w is accepted by NFA N if $\delta_N^*(s, w) \cap A \neq \emptyset$.

Definition

The language $L(N)$ accepted by a NFA $N = (Q, \Sigma, \delta, s, A)$ is

$$\{w \in \Sigma^* \mid \delta^*(s, w) \cap A \neq \emptyset\}.$$

Important: Formal definition of the language of NFA above uses δ^* and not δ. As such, one does not need to include ε-transitions closure when specifying δ, since δ^* takes care of that.

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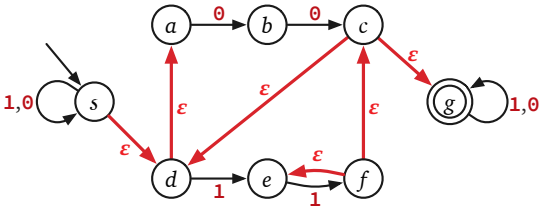

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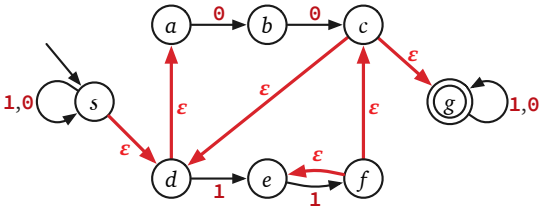

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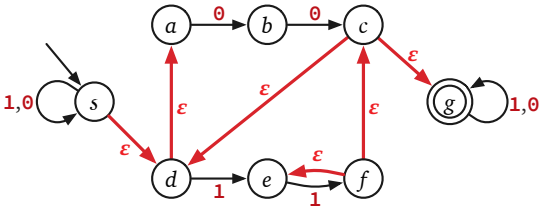

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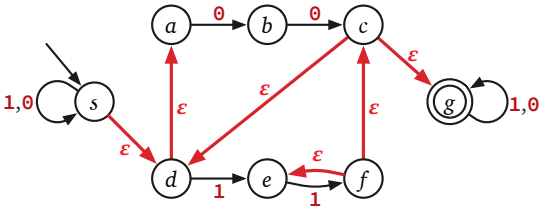

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Another definition of computation

Definition

$q \xrightarrow{w}_N p$: State p of NFA N is **reachable** from q on $w \iff$ there exists a sequence of states $r_0, r_1, \ldots, r_k$ and a sequence $x_1, x_2, \ldots, x_k$ where $x_i \in \Sigma \cup \{\varepsilon\}$, for each i, such that:

- $r_0 = q$,
- for each i, $r_{i+1} \in \delta^*(r_i, x_{i+1})$,
- $r_k = p$, and
- $w = x_1 x_2 x_3 \cdots x_k$.

Definition

$$\delta_N^*(q, w) = \left\{ p \in Q \;\middle|\; q \xrightarrow{w}_N p \right\}.$$

Why non-determinism?

- Non-determinism adds power to the model; richer programming language and hence (much) easier to "design" programs
- Fundamental in **theory** to prove many theorems
- Very important in **practice** directly and indirectly
- Many deep connections to various fields in Computer Science and Mathematics

Many interpretations of non-determinism. Hard to understand at the outset. Get used to it and then you will appreciate it slowly.

THE END

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(for now)