3.1.3
Extending the transition function to strings

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Useful to have notation to specify the unique state that M will reach from q on reading string w

Transition function $\delta^* : Q \times \Sigma^* \to Q$ defined inductively as follows:

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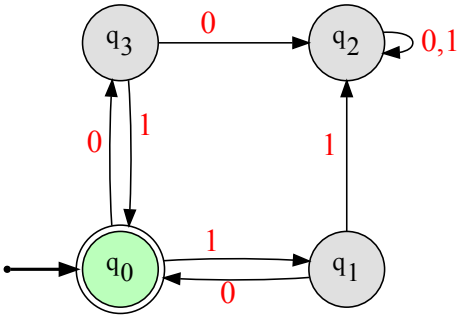
Formal definition of language accepted by M

Definition

The language $L(M)$ accepted by a DFA $M = (Q, \Sigma, \delta, s, A)$ is

$$\{w \in \Sigma^* \mid \delta^*(s, w) \in A\}.$$

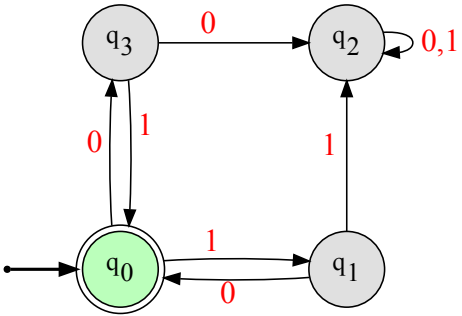
Example



What is:

- $\delta^*(q_1, \epsilon)$
- $\delta^*(q_0, 1011)$
- $\delta^*(q_1, 010)$
- $\delta^*(q_4, 10)$
- So what is $L(M)$??????

Example



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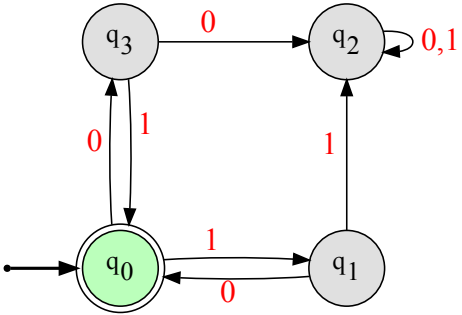

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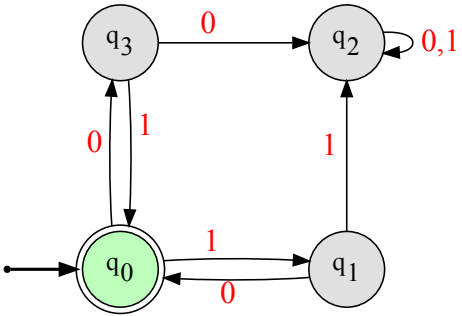

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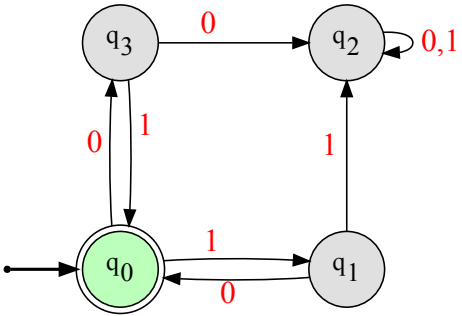

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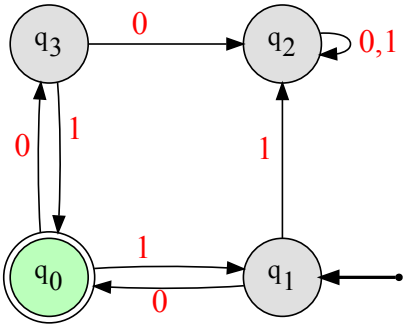
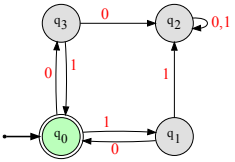
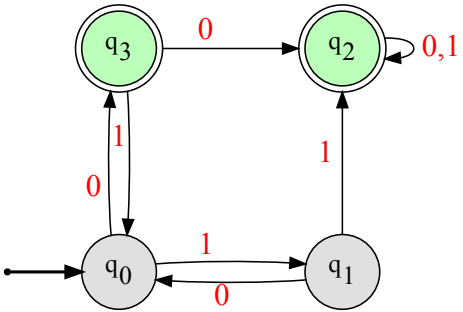


Old:

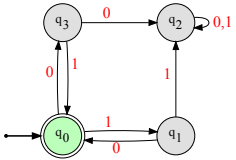


- What is $L(M)$ if start state is changed to q_1?

Example continued

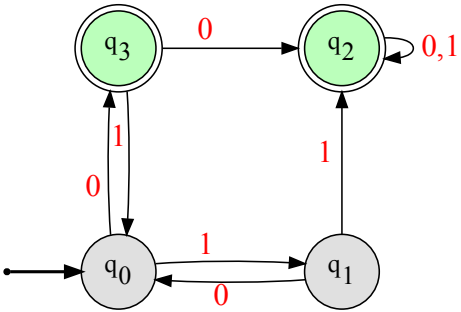


Old version:

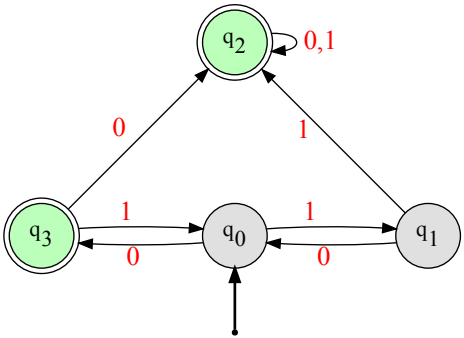


- What is $L(M)$ if final/accept states are set to $\{q_2, q_3\}$ instead of $\{q_0\}$?

Example continued



Redraw:



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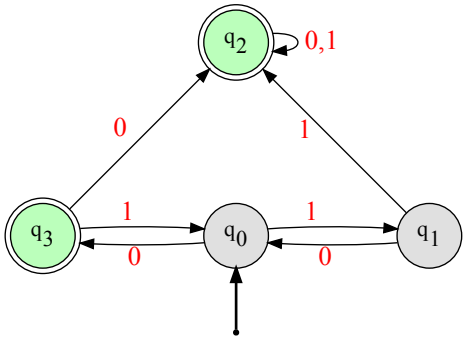
Example continued



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Advantages of formal specification

- Necessary for proofs
- Necessary to specify abstractly for class of languages

Exercise: Prove by induction that for any two strings u, v, any state q, $\delta^*(q, uv) = \delta^*(\delta^*(q, u), v)$.

THE END

...

(for now)