HW 4

Due on **Wednesday, September 21, 2022 at 9am CST**

CS/ECE 374A: Intro. Algorithms & Models of Computation, Fall 2022 Version: **1.2**

Submission instructions as in previous homeworks.

7 (100 PTS.) Fooling sets revisited.

(You can not use the Myhill-Nerode theorem in solving this exercise, since this exercise is the MN theorem.) Let L be a regular language over $\Sigma = \{0, 1\}$. Let $F = \{f_1, \ldots, f_k\}$ be a maximum cardinality fooling set for L (F must be finite, as otherwise L would *not* be a regular language).

7.A. (40 PTS.) Prove that for any string $x \in \Sigma^*$, there exists a unique string $\alpha(x) \in F$, such that x and $\alpha(x)$ are indistinguishable for L.

7.B. (30 PTS.) Prove that for any string $x \in \Sigma^*$, and $c \in \Sigma$, we have $\alpha(xc) = \alpha(\alpha(x)c)$.

7.C. (30 PTS.) Consider the following DFA: $M = (F, \Sigma, \delta, s, A)$, where

$$\forall f \in F, c \in \Sigma \qquad \delta(f, c) = \alpha(fc),$$

$s = \alpha(\varepsilon)$, and $A = F \cap L$.

Prove that $L(M) = L$.

[Hint: First prove that for any $x \in \Sigma^*$, we have $\delta^*(s, x) = \alpha(x)$.]

8 (100 PTS.) Context is everything.

Give a context-free grammar (CFG) for each of the following languages. You must provide explanation for how your grammar works, by describing in English what is generated by each non-terminal. (Formal proofs of correctness are not required.)

8.A. (30 PTS.) $L_1 = \left\{ 0^i 1^j 0^k \;\middle|\; i = j + k \text{ and } i, j, k \geq 0 \right\}$.

8.B. (30 PTS.) $L_2 = \{ x(110)^n x(111)^n \mid x \in \{0, 1\}, n \geq 1 \}$.

8.C. (40 PTS.) $L_3 = \left\{ 1^i 0^j 1^k 0^\ell \;\middle|\; i + j = k + \ell,\; i, j, k, \ell \geq 0 \right\}$.