For each statement below, check “Yes” if the statement is ALWAYS true and “No” otherwise, and give a brief explanation of your answer.

(a) Every integer in the empty set is prime.

(b) The language \( \{0^m 1^n \mid m + n \leq 374 \} \) is regular.

(c) The language \( \{0^m 1^n \mid m - n \leq 374 \} \) is regular.

(d) For all languages \( L \), the language \( L^* \) is regular.

(e) For all languages \( L \), the language \( L^* \) is infinite.

(f) For all languages \( L \subset \Sigma^* \), if \( L \) can be represented by a regular expression, then \( \Sigma^* \setminus L \) is recognized by a DFA.

(g) For all languages \( L \) and \( L' \), if \( L \cap L' = \emptyset \) and \( L' \) is not regular, then \( L \) is regular.

(h) Every regular language is recognized by a DFA with exactly one accepting state.

(i) Every regular language is recognized by an NFA with exactly one accepting state.

(j) Every language is either regular or context-free.
For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, either prove that the language is regular or prove that the language is not regular. *Exactly one of these two languages is regular.* Both of these languages contain the string $00110100000110100$.

1. $\{0^n w 0^n \mid w \in \Sigma^+ \text{ and } n > 0\}$

   $= \{0^n w 0^n \mid w \in \Sigma^+ \} \cup \{0^n w 0^n \mid w \in \Sigma^+ \}$

   $= \emptyset (0+1)(0+1)^+ \emptyset$

   *regular*

   **NFA:**

   ![NFA Diagram]

2. $\{w^nw \mid w \in \Sigma^+ \text{ and } n > 0\}$

   If $w \in \Sigma^*$ then decamp is unique.

   Let $F = \Sigma^*$

   Pick two strings $x, y \in F$.

   Then $x = 1^i$ and $y = 1^j$ for some $i, j$.

   Let $z = 01i$.

   Then $xz = 1^i01i \in L$

   $yz = 1^i01i \notin L$

   $F$ is infinite fooling set so $L$ not regular.
The parity of a bit-string $w$ is 0 if $w$ has an even number of 1s, and 1 if $w$ has an odd number of 1s. For example:

$$\text{parity}(e) = 0 \quad \text{parity}(0010100) = 0 \quad \text{parity}(00101110100) = 1$$

(a) Give a self-contained, formal, recursive definition of the parity function. (In particular, do not refer to # or other functions defined in class.)

$$\text{parity}(w) = \begin{cases} 0 & \text{if } w = \varepsilon \\ \text{parity}(x) & \text{if } w = 0x \\ 1 - \text{parity}(x) & \text{if } w = 1x \end{cases}$$

(b) Let $L$ be an arbitrary regular language. Prove that the language $\text{OddParity}(L) := \{w \in L \mid \text{parity}(w) = 1\}$ is also regular.

$$\text{OP}(L) = L \cap 0^*1(0^*10^*1)^*0^*$$

Build prod construction of DFA for $L$ and

$$\rightarrow 0 \xrightarrow{1} 1 \xrightarrow{0} 0 \xrightarrow{1} 2$$

where $A' = A_1 \times A_2$

(c) Let $L$ be an arbitrary regular language. Prove that the language $\text{AddParity}(L) := \{\text{parity}(w) \cdot w \mid w \in L\}$ is also regular.

Let $EPL(L) = L \cap (0^*10^*1)^*0 = L \setminus \text{OP}(L)$

Then $\text{AddParity}(L) = 0 \cdot EPL(L) + 1 \cdot \text{OP}(L)$

$$\text{AddParity}(L) = EPL((0+1) \cdot L)$$

Let $M = (Q, \Sigma, A, s)$ be DFA for $L$. We build an NFA $M' = (Q', \Sigma, A', s')$ for $\text{AddParity}(L)$ as follows:

$$Q' = (Q \times \{0, 1\}) \cup \{s'\}$$

$s' = \text{new state}$

$\delta(s', a) = (s, a)$

$\delta((q, p), a) = (\delta(q, a), p \oplus a)$

[Hint: Yes, you have enough room.]
For each of the following languages $L$, give a regular expression that represents $L$ and describe a DFA that recognizes $L$. You do not need to prove that your answers are correct.

(a) All strings in $(0 + 1)^*$ that do not contain the substring $0110$.

(b) All strings in $0^*10^*$ whose length is a multiple of 3.
For any string $w \in \{0, 1\}^*$, let $\text{obliviate}(w)$ denote the string obtained from $w$ by removing every 1. For example:

- $\text{obliviate}(\varepsilon) = \varepsilon$
- $\text{obliviate}(000000) = 000000$
- $\text{obliviate}(111111) = \varepsilon$
- $\text{obliviate}(01001101) = 00000$

Let $L$ be an arbitrary regular language.

1. **Prove** that the language $\text{OBLIVIATE}(L) = \{\text{obliviate}(w) \mid w \in L\}$ is regular.

   Let $M = (Q, S, A, S)$ be DFA for $L$.

   Build DFA for $\text{OBLIVIATE}(L)$ as follows:

   $Q' = Q$
   $S' = S$
   $A' = A$
   $s'(q_0, 0) = s(q_0, 0)$
   $s'(q_0, 1) = q$

   **SWAP!**

2. **Prove** that the language $\text{UNOBLIVIATE}(L) = \{w \in \{0, 1\}^* \mid \text{obliviate}(w) \in L\}$ is regular.

   Let $M = (Q, s, A, S)$ be DFA for $L$.

   Build NFA for $\text{UNOBLIVIATE}(L)$ as follows:

   $Q' = Q$
   $S' = S$
   $A' = A$
   $s'(q_0, 0) = s(q_0, 0)$
   $s'(q_0, 1) = \emptyset$
   $s'(q, 1) = s(q, 1)$

   **SWAP!**