

LANGUAGE TRANSFORMATIONS.

Q: let $T(L) = \{ w \mid ww^R \in L \}$

Prove that for any regular language L ,

$T(L)$ is also regular.

Proof: Given a DFA $M = (\mathcal{Q}, s, A, S)$ for L , we will build an NFA $M' = (\mathcal{Q}', s', A', S')$.

Goal: M' should accept w iff ww^R is accepted by M .

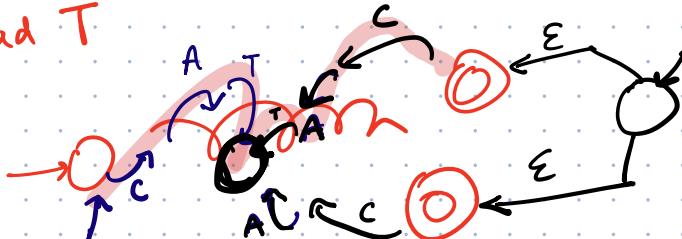
Eg. $w = CAT$ should be accepted by M'

iff $CATTAC$ is accepted by M

M' : reads $\xrightarrow{C} C$, then A , then T .

once M' has read T

arbitrary M



M' is a product construction

of DFA M and NFA $\underline{M^R}$ recall:

[Run both M and M^R in parallel, meet in the middle]

$$\begin{aligned} S^R(s^R, \epsilon) \\ = \{q \mid q \in A\} \end{aligned}$$

$$Q' = Q \times (Q \cup \{s^R\})$$

$$S' = (s, s^R)$$

$$A' = \{(q, q) \mid q \in Q\}$$

~~$S'(q_1, q_2, a) = (S(q_1, a), S^R(q_2, a))$~~

$$S'((s, s^R), \epsilon) = \{(s, q) \mid q \in A\}$$

(essentially $S'((s, s^R), \epsilon) = (S(s, \epsilon), S^R(s, \epsilon))$)

$$S'((q, r), \epsilon) = \emptyset$$

$$S^R(r, a) = \{p \mid S(p, a) = r\}$$

$$S'((q, r), a) = (S(q, a), \underline{S^R(r, a)})$$

$$= \{(S(q, a), p) \mid S(p, a) = r\}$$

$$S'((q, s^R), a) = \emptyset. \quad \forall q \in Q.$$