1. Let \( \langle M \rangle \) denote the encoding of a Turing machine \( M \) (or if you prefer, the Python source code for the executable code \( M \)). Recall that \( w^R \) denotes the reversal of string \( w \). Prove that the following language is undecidable.

\[
\text{SelfRevAccept} := \{ \langle M \rangle \mid M \text{ accepts the string } \langle M \rangle^R \}
\]

Note that Rice's theorem does not apply to this language.

2. Let \( M \) be a Turing machine, let \( w \) be a string, and let \( s \) be an integer. We say that \( M \) accepts \( w \) in space \( s \) if, given \( w \) as input, \( M \) accesses at most the first \( s \) cells on its tape and eventually accepts. (If you prefer to think in terms of programs instead of Turing machines, “space” is how much memory your program needs to run correctly.)

Prove that the following language is undecidable:

\[
\text{SomeSquareSpace} = \{ \langle M \rangle \mid M \text{ accepts at least one string } w \text{ in space } |w|^2 \}
\]

Note that Rice's theorem does not apply to this language.

[Hint: The only thing you actually need to know about Turing machines for this problem is that they consume a resource called “space”.]

3. Prove that the following language is undecidable:

\[
\text{Picky} = \{ \langle M \rangle \mid M \text{ accepts at least one input string and } M \text{ rejects at least one input string} \}
\]

Note that Rice's theorem does not apply to this language.