For each statement below, check “Yes” if the statement is **ALWAYS** true and “No” otherwise, and give a **brief** explanation of your answer.

(a) Every integer in the empty set is prime.
    
    Yes  No

(b) The language \( \{0^m 1^n \mid m + n \leq 374\} \) is regular.
    
    Yes  No

(c) The language \( \{0^m 1^n \mid m - n \leq 374\} \) is regular.
    
    Yes  No

(d) For all languages \( L \), the language \( L^* \) is regular.
    
    Yes  No

(e) For all languages \( L \), the language \( L^* \) is infinite.
    
    Yes  No

(f) For all languages \( L \subseteq \Sigma^* \), if \( L \) can be represented by a regular expression, then \( \Sigma^* \setminus L \) is recognized by a DFA.
    
    Yes  No

(g) For all languages \( L \) and \( L' \), if \( L \cap L' = \emptyset \) and \( L' \) is not regular, then \( L \) is regular.
    
    Yes  No

(h) Every regular language is recognized by a DFA with exactly one accepting state.
    
    Yes  No

(i) Every regular language is recognized by an NFA with exactly one accepting state.
    
    Yes  No

(j) Every language is either regular or context-free.
    
    Yes  No
For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, either prove that the language is regular or prove that the language is not regular. Exactly one of these two languages is regular. Both of these languages contain the string $0011010000110100$.

1. $\{0^n1^n \mid w \in \Sigma^+ \text{ and } n > 0\}$

2. $\{w^n1^n \mid w \in \Sigma^+ \text{ and } n > 0\}$
The *parity* of a bit-string $w$ is 0 if $w$ has an even number of 1s, and 1 if $w$ has an odd number of 1s. For example:

\[
\text{parity}(\epsilon) = 0 \quad \text{parity}(0010100) = 0 \quad \text{parity}(00101110100) = 1
\]

(a) Give a self-contained, formal, recursive definition of the *parity* function. (In particular, do *not* refer to # or other functions defined in class.)

(b) Let $L$ be an arbitrary regular language. Prove that the language $\text{OddParity}(L) := \{w \in L | \text{parity}(w) = 1\}$ is also regular.

(c) Let $L$ be an arbitrary regular language. Prove that the language $\text{AddParity}(L) := \{\text{parity}(w) \cdot w | w \in L\}$ is also regular.

[Hint: Yes, you have enough room.]
For each of the following languages $L$, give a regular expression that represents $L$ and describe a DFA that recognizes $L$. You do not need to prove that your answers are correct.

(a) All strings in $(0 + 1)^*$ that do not contain the substring $0110$.

(b) All strings in $0^*10^*$ whose length is a multiple of 3.
For any string $w \in \{0, 1\}^*$, let $\text{obliviate}(w)$ denote the string obtained from $w$ by removing every 1. For example:

\[
\begin{align*}
\text{obliviate}(\epsilon) &= \epsilon \\
\text{obliviate}(000000) &= 000000 \\
\text{obliviate}(111111) &= \epsilon \\
\text{obliviate}(01001101) &= 00000
\end{align*}
\]

Let $L$ be an arbitrary regular language.

1. **Prove** that the language $\text{OBLIVIATE}(L) = \{\text{obliviate}(w) \mid w \in L\}$ is regular.

2. **Prove** that the language $\text{UNOBLIVIATE}(L) = \{w \in \{0, 1\}^* \mid \text{obliviate}(w) \in L\}$ is regular.