Lecture 23 Scribble

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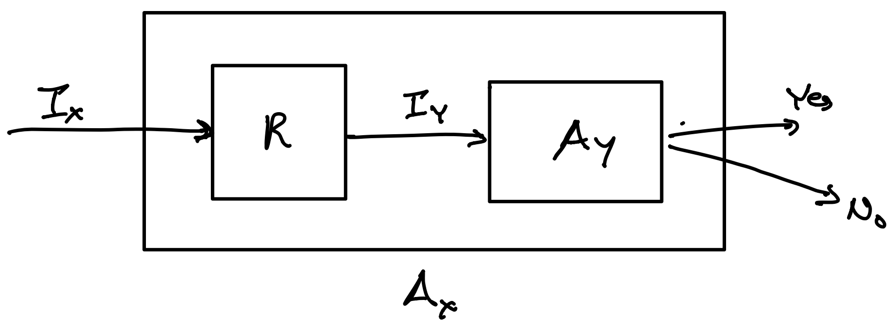
Topics :- Reductions
- Cliques / Independent Sets
- SAT

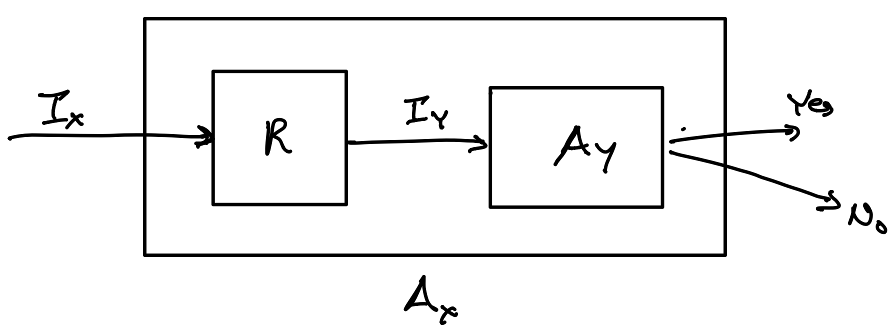
Two Problems $X \leq_p Y$



A_X = finding shortest path from s to t Djikstra's

A_Y = finding all shortest paths Bellman-Ford

Djikstra's $\leq$ Bellman-Ford



easy = has a polynomial time deterministic solution

hard =

$X \leq_p Y$ (1) X is no harder than Y

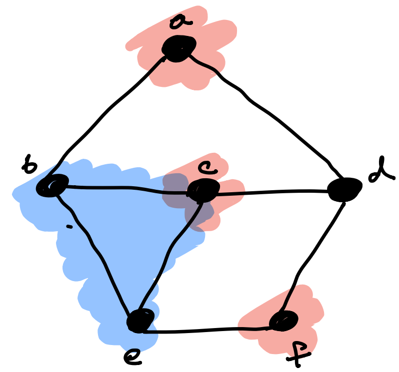
(2) If X is hard, Y should also be hard
Y is atleast as hard as X

Example: Clique / Independent Set

Independent Set: $I \subseteq V$ s.t. no $u, v \in I$ are connected by an edge

Clique: $C \subseteq V$ s.t. $\{u, v\} \in E$ for all $u, v \in C$

G(V,E)

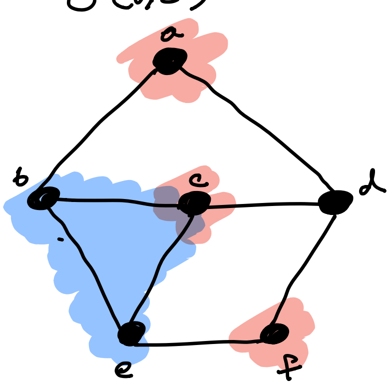


Given $G(V,E)$ and an integer k:

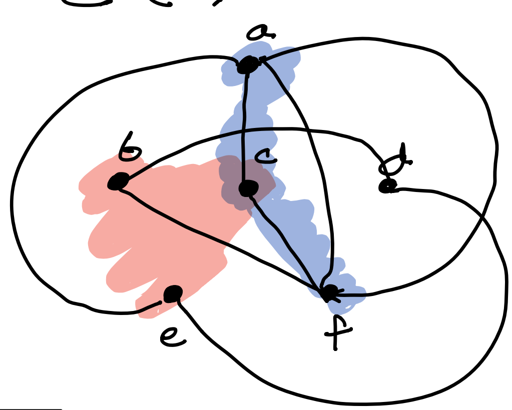
- Does G have an $|I.S| \geq k$? MAX IND SET
- Does G have a clique, $|C| \geq k$? MAX CLIQUE

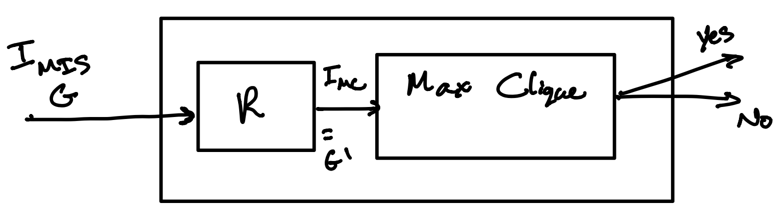
A_y = Solves MAX CLIQUE

G(V,E)



G'(V, E')



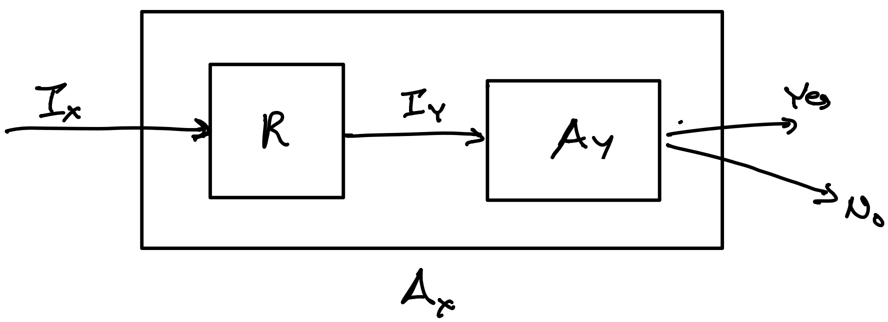


Max IS

R = finding the edge complement of the graph

$O(V^2)$

Max IS $\leq_p$ Max Clique   Max IS $\simeq_p$ Max Clique



$X \leq_p Y$

If Y is hard is X also hard? FALSE

If X is known to be "hard" then Y does not have a efficient algorithm. TRUE

Want to prove HW P # 4(x) is hard

Reduce a hard problem to X.
(IS or Clique)

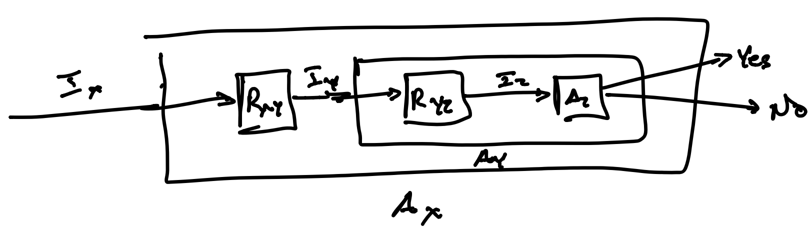
If Y has a polynomial time algorithm then it implies X has a polynomial time algorithm. Depends

If R is polynomial then true

If R is "hard" then false.

- $X \leq_p Y$ implies $Y \leq_p X$? FALSE

If $X \leq_p Y$ and $Y \leq_p Z$ then $X \leq_p Z$? TRUE



SAT Problems : (CNF, SAT, 3CNF, 3SAT)

$X = \{x_1, x_2, \ldots, x_N\}$ $\vee \equiv$ or $(+)$ $\wedge \equiv$ and $(\cdot)$

$$\Big[(\underset{\text{literal}}{x_1} \vee x_2 \vee \bar{x}_4) \wedge \underbrace{(x_2 \vee x_{\bar{3}})}_{\text{clause}} \wedge x_5\Big]$$

$\hookrightarrow$ conjunctive normal form

Is there a assignment to the boolean variables s.t. the expression is true.

$x_1 x_2 + \bar{x}_3 x_4 x_5 + x_1 \bar{x}_3 \ldots$

$\hookrightarrow$ disjunctive normal form

3CNF : $(x_1 \vee x_2 \vee \bar{x}_4) \wedge (x_3 \vee x_2 \vee x_8) \wedge \ldots$

Every boolean expression can be expressed as 3CNF

ab

$a \wedge b \equiv (\bar{a} \vee \bar{b}) \wedge (\bar{a} \vee b) \wedge (a \vee \bar{b})$

a	b	f
0	0	0
0	1	0
1	0	0
1	1	1

Problem : SAT (Satisfiability) (NP)

Given a CNF formula is there an assignment which results in the formula as evaluating true.

Every CNF expression can be reduced to 3CNF

- if clause > 3 literals $(a \vee b \vee c) \Rightarrow (a \vee b \vee z) \wedge (c \vee \bar{z})$

X / Y are boolean expressions if $(X \vee Y)$ are SAT

$(a \vee b \vee c \vee d) \Rightarrow (a \vee b \vee z) \wedge (c \vee d \vee \bar{z})$ then $(X \vee z) \wedge (Y \vee \bar{z})$ are SAT

- if clause has 2 literal

$$(a \vee b) \Longrightarrow (a \vee b \vee u) \wedge (a \vee b \vee \bar{u})$$

- if clause has 1 literal

$$(a) \Longrightarrow (a \vee u \vee v) \wedge (a \vee \bar{u} \vee v) \wedge (a \vee u \vee \bar{v}) \wedge (a \vee \bar{u} \vee \bar{v})$$