24.3.2

Problems related to graph coloring
Register allocation during compilation

1. When a compiler generates the assembly/VM code it needs to allocation registers to values being handled.

2. Need to make sure registers are not in conflict.

3. Build a conflict graph.

4. Color the conflict graph.

5. Every color is a register.

6. If not enough registers, then use memory/stack to store values.

7. CISC v.s. RISC.
Register allocation during compilation

1. When a compiler generates the assembly/VM code it needs to allocation registers to values being handled.
2. Need to make sure registers are not in conflict.
3. Build a conflict graph.
4. Color the conflict graph.
5. Every color is a register.
6. If not enough registers, then use memory/stack to store values.
7. CISC v.s. RISC.
## Graph Coloring and Register Allocation

### Register Allocation

Assign variables to (at most) $k$ registers such that variables needed at the same time are not assigned to the same register.

### Interference Graph

Vertices are variables, and there is an edge between two vertices, if the two variables are “live” at the same time.

### Observations

- [Chaitin] Register allocation problem is equivalent to coloring the interference graph with $k$ colors.
- Moreover, $3$-COLOR $\leq_P k$-Register Allocation, for any $k \geq 3$.
Class Room Scheduling

1. Given $n$ classes and their meeting times, are $k$ rooms sufficient?
2. Reduce to Graph $k$-Coloring problem
3. Create graph $G$
   - a node $v_i$ for each class $i$
   - an edge between $v_i$ and $v_j$ if classes $i$ and $j$ conflict
4. Exercise: $G$ is $k$-colorable $\iff$ $k$ rooms are sufficient.
Class Room Scheduling

1. Given \( n \) classes and their meeting times, are \( k \) rooms sufficient?
2. Reduce to Graph \( k \)-Coloring problem
3. Create graph \( G \)
   - a node \( v_i \) for each class \( i \)
   - an edge between \( v_i \) and \( v_j \) if classes \( i \) and \( j \) conflict
4. Exercise: \( G \) is \( k \)-colorable \( \iff \) \( k \) rooms are sufficient.
Class Room Scheduling

1. Given \( n \) classes and their meeting times, are \( k \) rooms sufficient?
2. Reduce to Graph \( k \)-Coloring problem
3. Create graph \( G \)
   - a node \( v_i \) for each class \( i \)
   - an edge between \( v_i \) and \( v_j \) if classes \( i \) and \( j \) conflict
4. Exercise: \( G \) is \( k \)-colorable \( \iff \) \( k \) rooms are sufficient.
Class Room Scheduling

1. Given \( n \) classes and their meeting times, are \( k \) rooms sufficient?
2. Reduce to Graph \( k \)-Coloring problem
3. Create graph \( G \)
   - a node \( v_i \) for each class \( i \)
   - an edge between \( v_i \) and \( v_j \) if classes \( i \) and \( j \) conflict
4. Exercise: \( G \) is \( k \)-colorable ⇐⇒ \( k \) rooms are sufficient.
Frequency Assignments in Cellular Networks

1. Cellular telephone systems that use Frequency Division Multiple Access (FDMA) (example: GSM in Europe and Asia and AT&T in USA)
   ▶ Breakup a frequency range $[a, b]$ into disjoint bands of frequencies $[a_0, b_0], [a_1, b_1], \ldots, [a_k, b_k]$
   ▶ Each cell phone tower (simplifying) gets one band
   ▶ Constraint: nearby towers cannot be assigned same band, otherwise signals will interference

2. Problem: given $k$ bands and some region with $n$ towers, is there a way to assign the bands to avoid interference?

3. Can reduce to $k$-coloring by creating interference/conflict graph on towers.
Frequency Assignments in Cellular Networks

1. Cellular telephone systems that use Frequency Division Multiple Access (FDMA) (example: GSM in Europe and Asia and AT&T in USA)
   - Breakup a frequency range $[a, b]$ into disjoint bands of frequencies $[a_0, b_0], [a_1, b_1], \ldots, [a_k, b_k]$
   - Each cell phone tower (simplifying) gets one band
   - Constraint: nearby towers cannot be assigned same band, otherwise signals will interfere

2. Problem: given $k$ bands and some region with $n$ towers, is there a way to assign the bands to avoid interference?

3. Can reduce to $k$-coloring by creating interference/conflict graph on towers.
Frequency Assignments in Cellular Networks

1. Cellular telephone systems that use Frequency Division Multiple Access (FDMA) (example: GSM in Europe and Asia and AT&T in USA)
   ▶ Breakup a frequency range \([a, b]\) into disjoint bands of frequencies \([a_0, b_0], [a_1, b_1], \ldots, [a_k, b_k]\)
   ▶ Each cell phone tower (simplifying) gets one band
   ▶ Constraint: nearby towers cannot be assigned same band, otherwise signals will interfere

2. Problem: given \(k\) bands and some region with \(n\) towers, is there a way to assign the bands to avoid interference?

3. Can reduce to \(k\)-coloring by creating interference/conflict graph on towers.
THE END

... (for now)