24.3

NP-Completeness of Graph Coloring
24.3.1
The coloring problem
Problem: Graph Coloring

Instance: \( G = (V, E) \): Undirected graph, integer \( k \).

Question: Can the vertices of the graph be colored using \( k \) colors so that vertices connected by an edge do not get the same color?
Graph 3-Coloring

Problem: 3 Coloring

Instance: $G = (V, E)$: Undirected graph.

Question: Can the vertices of the graph be colored using 3 colors so that vertices connected by an edge do not get the same color?
Graph 3-Coloring

Problem: 3 Coloring

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**Question:** Can the vertices of the graph be colored using 3 colors so that vertices connected by an edge do not get the same color?
Graph Coloring

1. **Observation**: If $G$ is colored with $k$ colors then each color class (nodes of same color) form an independent set in $G$.

2. $G$ can be partitioned into $k$ independent sets $\iff G$ is $k$-colorable.

3. Graph 2-Coloring can be decided in polynomial time.

4. $G$ is 2-colorable $\iff G$ is bipartite.

5. There is a linear time algorithm to check if $G$ is bipartite using **BFS** (we saw this earlier).
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THE END

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(for now)