23.4
Hamiltonian cycle in undirected graph
Hamiltonian Cycle

Problem 23.1.

Input  Given undirected graph $G = (V, E)$

Goal  Does $G$ have a Hamiltonian cycle? That is, is there a cycle that visits every vertex exactly one (except start and end vertex)?
Theorem 23.2. 

*Hamiltonian cycle problem for undirected graphs is NP-Complete.*

Proof.

- The problem is in **NP**; proof left as exercise.
- Hardness proved by reducing Directed Hamiltonian Cycle to this problem.
Reduction Sketch

**Goal:** Given directed graph $G$, need to construct undirected graph $G'$ such that $G$ has Hamiltonian Path iff $G'$ has Hamiltonian path

**Reduction**
- Replace each vertex $v$ by 3 vertices: $v_{in}$, $v$, and $v_{out}$
- A directed edge $(a, b)$ is replaced by edge $(a_{out}, b_{in})$
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![Diagram showing the reduction process](image)
Hamiltonian cycle reduction

Undirected to directed case
Hamiltonian cycle reduction

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Undirected to directed case
Reduction: Wrap-up

- The reduction is polynomial time (exercise)
- The reduction is correct (exercise)
THE END

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(for now)