23.4
Hamiltonian cycle in undirected graph

Hamiltonian Cycle

Problem 23.1.

Input Given undirected graph $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E})$

Goal Does $\boldsymbol{G}$ have a Hamiltonian cycle? That is, is there a cycle that visits every vertex exactly one (except start and end vertex)?

NP-Completeness

Theorem 23.2.

Hamiltonian cycle *problem for* ***undirected*** *graphs is* **NP-Complete**.

Proof.

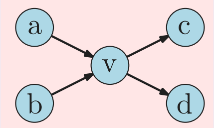
- The problem is in **NP**; proof left as exercise.
- Hardness proved by reducing Directed Hamiltonian Cycle to this problem $\square$

Reduction Sketch

Goal: Given directed graph G, need to construct undirected graph G' such that G has Hamiltonian Path iff G' has Hamiltonian path

Reduction

- Replace each vertex v by 3 vertices: v_{in}, v, and v_{out}
- A directed edge (a, b) is replaced by edge (a_{out}, b_{in})



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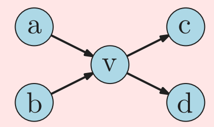

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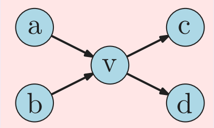

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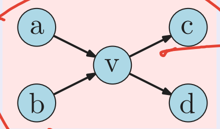
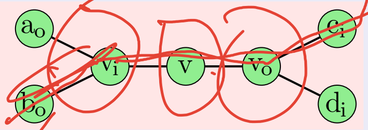
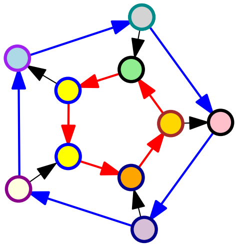


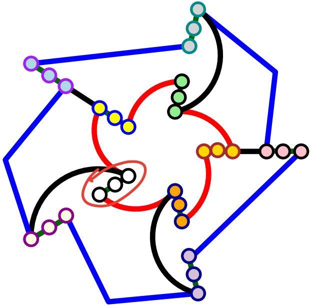


Hamiltonian cycle reduction

Undirected to directed case



$\Rightarrow$



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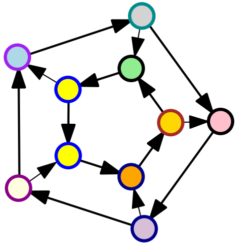

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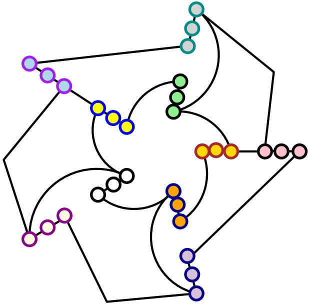

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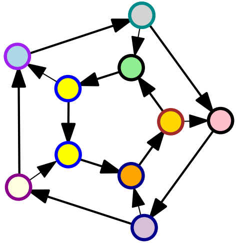

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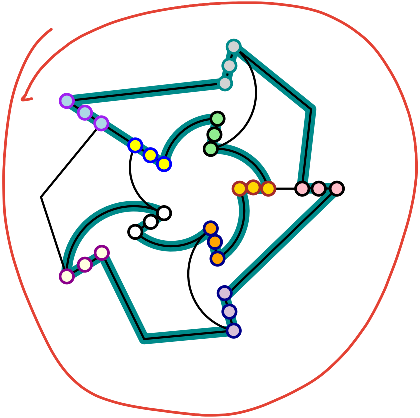

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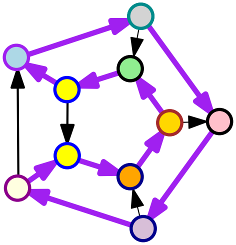

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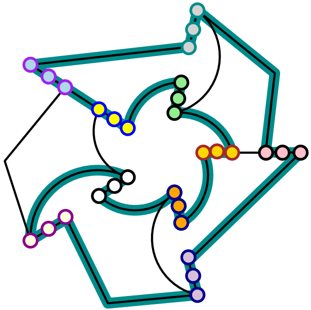

Reduction: Wrap-up

- The reduction is polynomial time (exercise)
- The reduction is correct (exercise)

THE END

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(for now)