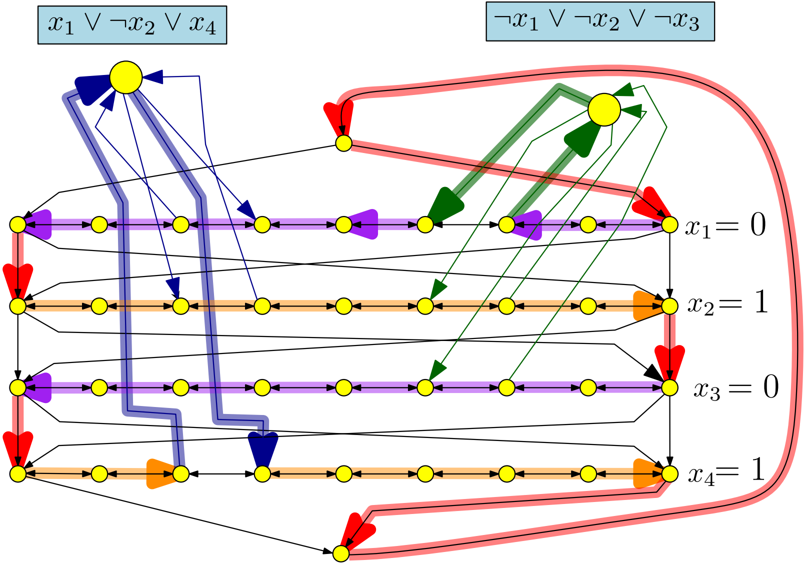
23.3.4

If there is a Hamiltonian cycle $\implies$ $\exists$satisfying assignment

Reduction: Hamiltonian cycle $\implies$ $\exists$ satisfying assignment

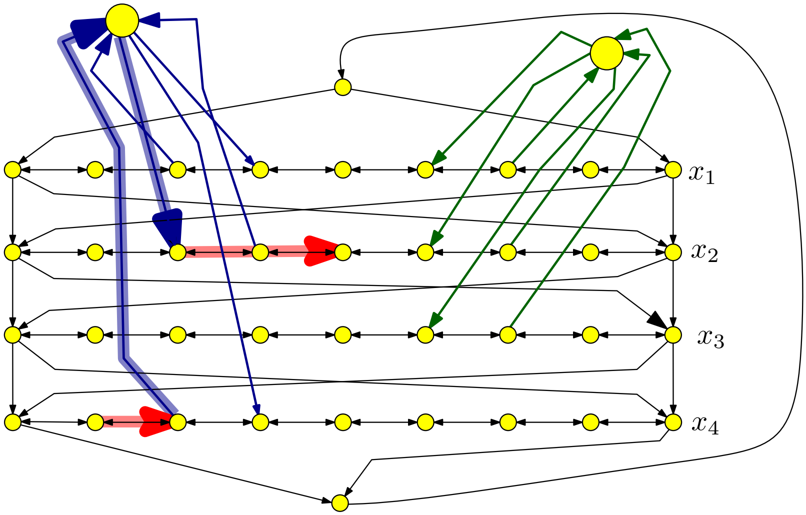
We are given a Hamiltonian cycle in $\boldsymbol{G_\varphi}$:



Want to extract satisfying assignment...

Reduction: Hamiltonian cycle $\implies$ $\exists$ satisfying assignment

No shenanigan: Hamiltonian cycle can not leave a row in the middle



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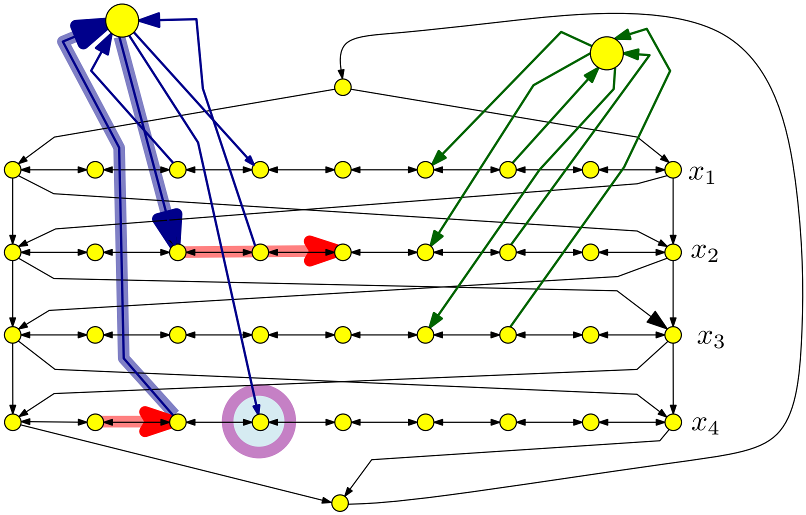

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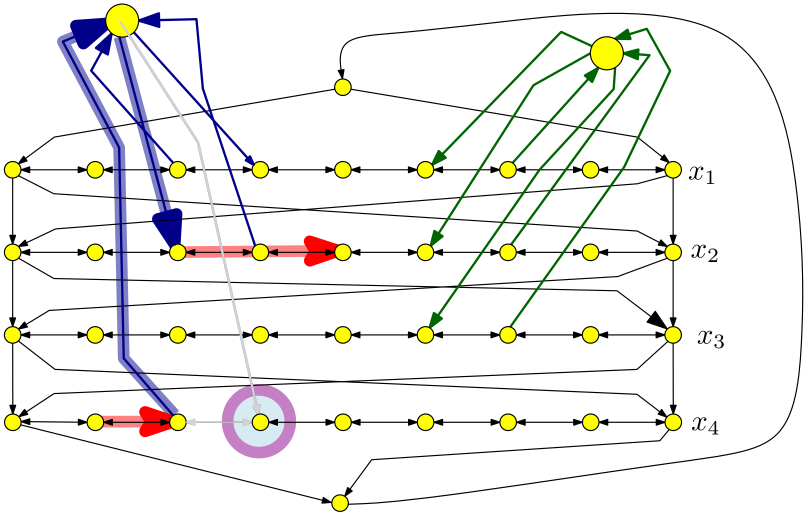

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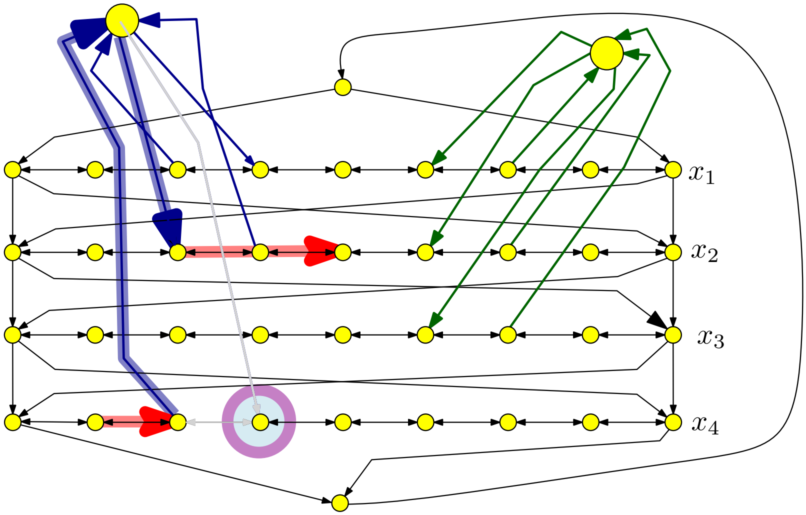

Conclude: Hamiltonian cycle must go through each row completely from left to right, or right to left. As such, can be interpreted as a valid assignment.

Hamiltonian Cycle $\Rightarrow$ Satisfying assignment

Suppose Π is a Hamiltonian cycle in G_φ

- If Π enters c_j (vertex for clause C_j) from vertex $3j$ on path i then it must leave the clause vertex on edge to $3j+1$ on the same path i
  - If not, then only unvisited neighbor of $3j+1$ on path i is $3j+2$
  - Thus, we don't have two unvisited neighbors (one to enter from, and the other to leave) to have a Hamiltonian Cycle
- Similarly, if Π enters c_j from vertex $3j+1$ on path i then it must leave the clause vertex c_j on edge to $3j$ on path i

Hamiltonian Cycle $\implies$ Satisfying assignment (contd)

- Thus, vertices visited immediately before and after C_i are connected by an edge
- We can remove c_j from cycle, and get Hamiltonian cycle in $G - c_j$
- Consider Hamiltonian cycle in $G - \{c_1, \ldots c_m\}$; it traverses each path in only one direction, which determines the truth assignment

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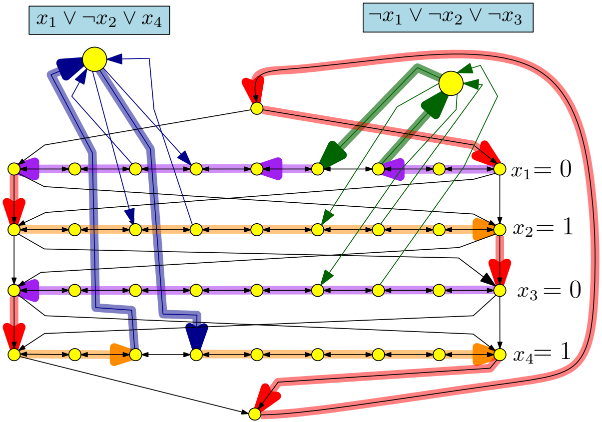

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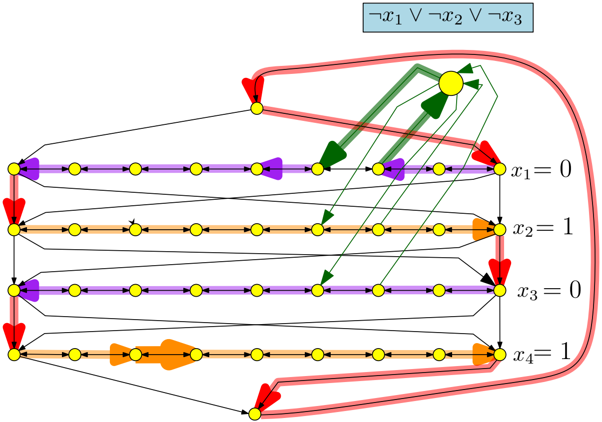

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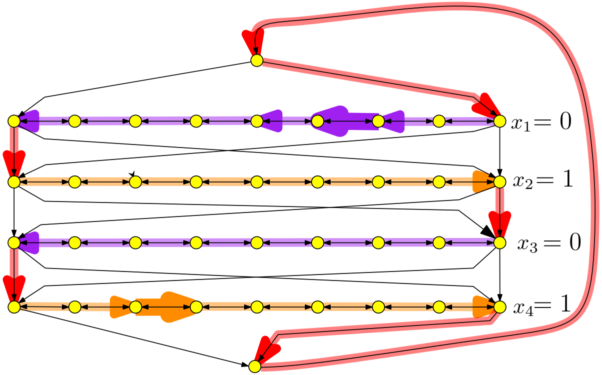

Correctness Proof

We just proved:

Lemma 23.2.

G_φ has a Hamiltonian cycle $\implies$ φ has a satisfying assignment α.

Lemma 23.3.

φ has a satisfying assignment iff G_φ has a Hamiltonian cycle.

Proof.

Follows from **Lemma 23.1** and **Lemma 23.2** . □

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Summary

What we did:

1. Showed that **Directed Hamiltonian Cycle** is in **NP**.
2. Provided a polynomial time reduction from **3SAT** to **Directed Hamiltonian Cycle**.
3. Proved that φ satisfiable $\iff$ G_φ is Hamiltonian.

Theorem 23.4.

The problem ***Hamiltonian Cycle*** *in directed graphs is* **NP-Complete***.*

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Theorem 23.4.

*The problem **Hamiltonian Cycle** in directed graphs is **NP-Complete**.*

THE END

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(for now)