

## 23.3.2

The reduction: Encoding the formula constraints

# 3SAT $\leq_P$ Directed Hamiltonian Cycle

**Input:**  $\varphi$  formula.

**Output:** Graph  $G_\varphi$ .

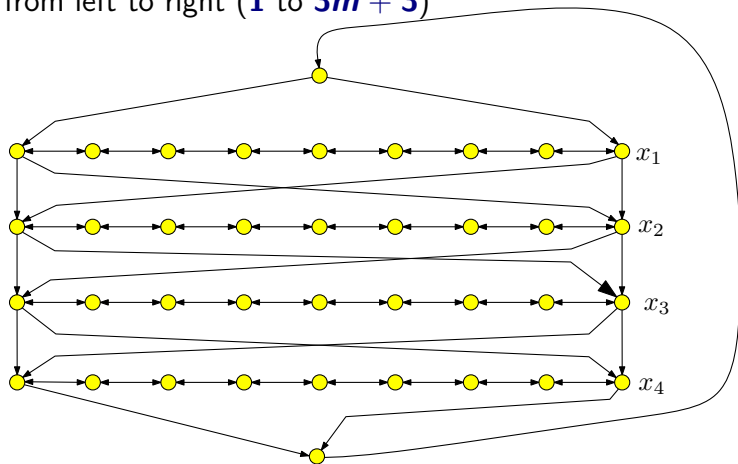
Saw: How to encode assignments...

Now need to encode constraints of  $\varphi$ .

# The reduction algorithm: Phase I

Converting  $\varphi$  to a graph

- ▶ Traverse path  $i$  from left to right iff  $x_i$  is set to true
- ▶ Each path has  $3(m + 1)$  nodes where  $m$  is number of clauses in  $\varphi$ ; nodes numbered from left to right ( $1$  to  $3m + 3$ )

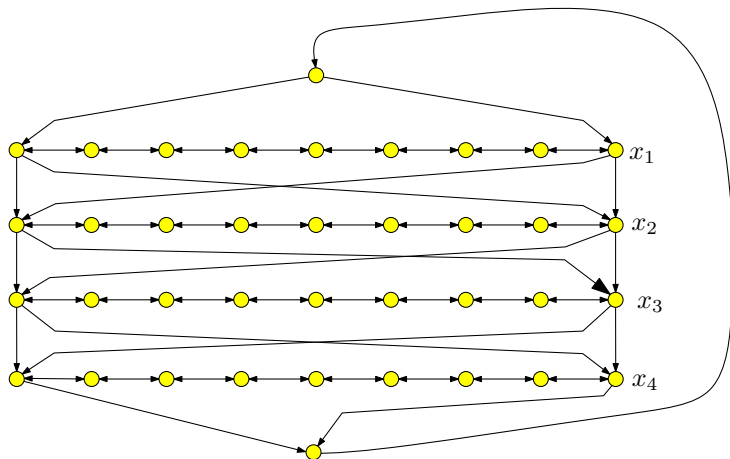


# The Reduction algorithm: Phase II

- ▶ Add vertex  $c_j$  for clause  $C_j$ .  $c_j$  has edge from vertex  $3j$  and to vertex  $3j + 1$  on path  $i$  if  $x_i$  appears in clause  $C_j$ , and has edge from vertex  $3j + 1$  and to vertex  $3j$  if  $\neg x_i$  appears in  $C_j$ .

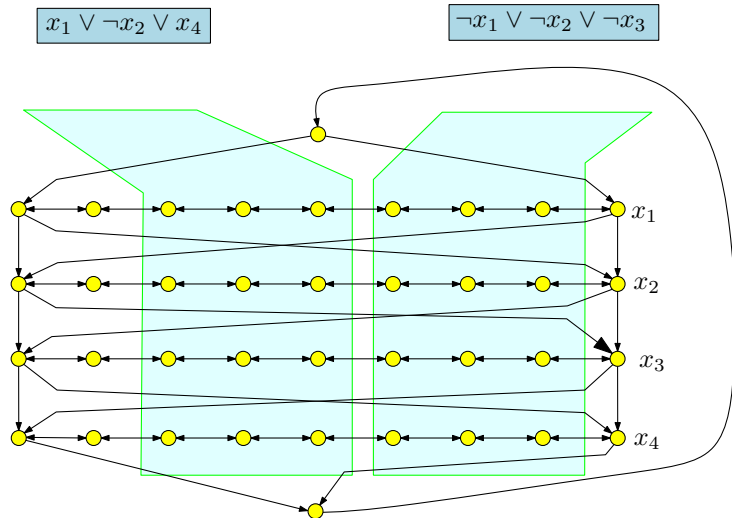
$$x_1 \vee \neg x_2 \vee x_4$$

$$\neg x_1 \vee \neg x_2 \vee \neg x_3$$



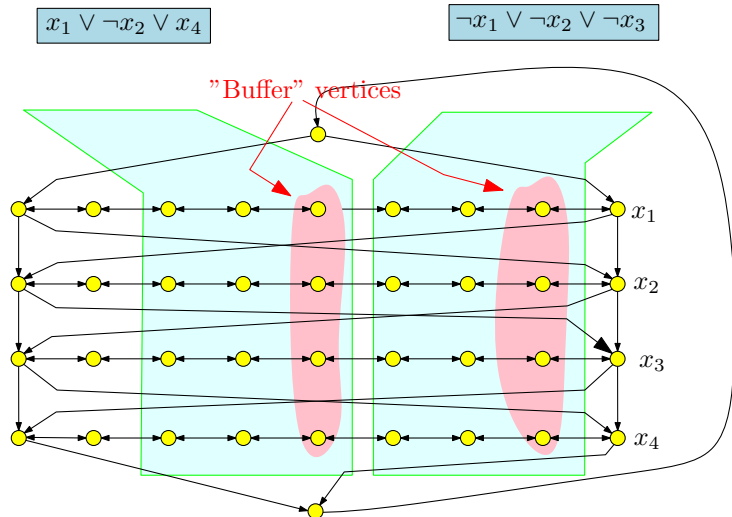
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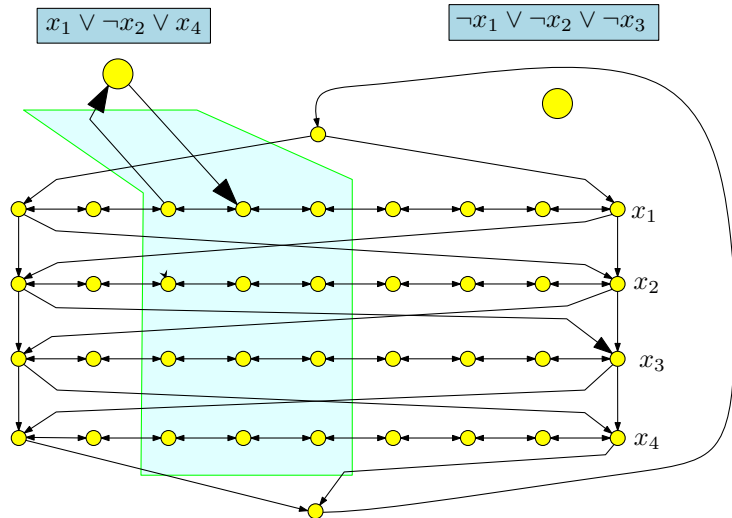
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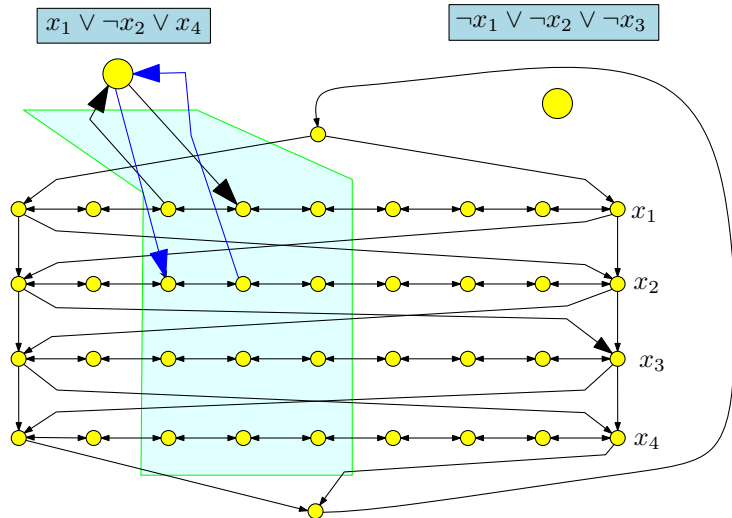
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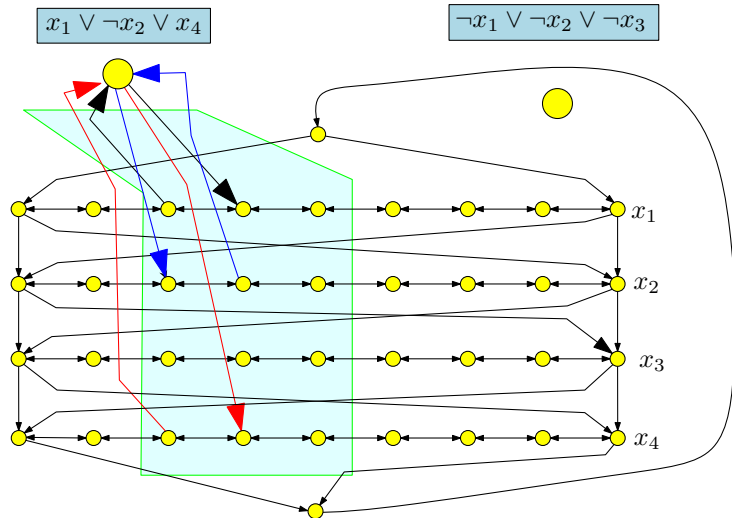
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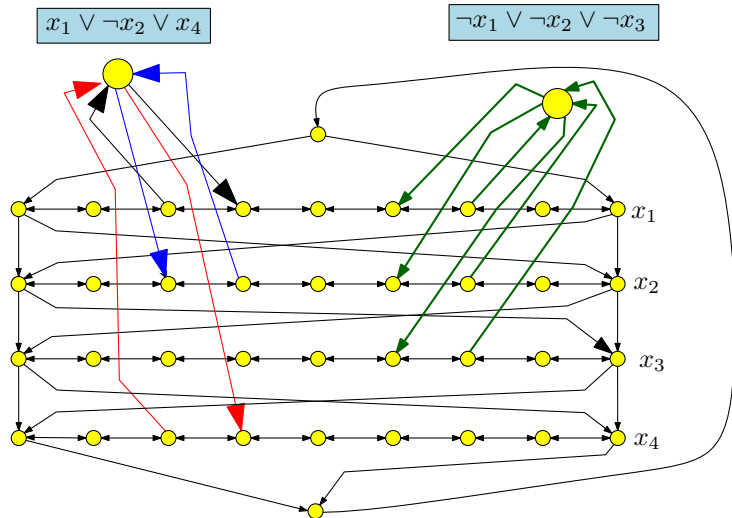
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**THE END**

...

**(for now)**