Polynomial Time Reductions

Lecture 21
Tuesday, November 17, 2020
21.1
A quick review: Polynomials
What is a polynomial

A **polynomial** is a function of the form:

\[ f(x) = \sum_{i=0}^{t} a_i x^i. \]

For our purposes, we can assume that \( a_i \geq 0 \), for all \( i \).
A term \( a_k x^t \) is a **monomial**.

The degree of \( f(x) \) is \( t \).
We have \( f(n) = O(n^t) \).
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The degree of the polynomial matters...
Polynomial time good, exponential time bad
Combining polynomials

**Lemma 21.1.**

If \( f(x) = \sum_{i=0}^{d} \alpha_i x^i \) is a polynomial of degree \( d \), and \( g(y) = \sum_{i=0}^{d'} \beta_i y^i \) is a polynomial of degree \( d' \), then \( g(f(x)) \) is a polynomial of degree \( d'd \).

**Proof.**

Observe that \((f(x))^2 = \sum_{i=0}^{d} \sum_{j=0}^{d} \alpha_i \alpha_j x^{i+j}\) is a polynomial of degree \(2d\), Arguing similarly, we have that \((f(x))^i\) is a polynomial of degree \(i \cdot d\). Thus

\[
g(f(x)) = \sum_{i=0}^{d'} \beta_i (f(x))^i
\]

is a sum of polynomials of degree \(0, d, 2d, \ldots, d \cdot d'\), which is a polynomial of degree \(d \cdot d'\) by collecting monomials of the same degree into a single monomial.

\[\square\]
THE END

... 

(for now)