20.4.2
The safe edges form the MST
Lemma 20.3.

Let $G$ be a connected graph with distinct edge costs, then the set of safe edges form a connected graph.

Proof.

1. Suppose not. Let $S$ be a connected component in the graph induced by the safe edges.
2. Consider the edges crossing $S$, there must be a safe edge among them since edge costs are distinct and so we must have picked it.
Safe Edges do not contain a cycle

Lemma 20.4.
Let $G$ be a connected graph with distinct edge costs, then the set of safe edges does not contain a cycle.

Proof.
Proposition 20.5: proved every edge in graph is either safe or unsafe. If $\exists$ cycle, then by definition the most expensive edge in the cycle is unsafe. Contradiction.
Lemma 20.4.

Let $G$ be a connected graph with distinct edge costs, then the set of safe edges does not contain a cycle.

Proof.

Assume false, and let $\pi$ a cycle made of safe edges.

$e$: Most expensive edge in the cycle $\pi$.

$C = (S, V \setminus S)$: Cut that $e$ is safe for.

$\pi$ must have at least two edges in $C$.

$f$: cheapest edge in $\pi \cap C$.

$e$ is not cheapest edge in $C$.

A contradiction.
Safe Edges do not contain a cycle

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Let $G$ be a connected graph with distinct edge costs, then the set of safe edges does not contain a cycle.

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A contradiction. $\square$
Safe Edges do not contain a cycle

Lemma 20.4.
Let $G$ be a connected graph with distinct edge costs, then the set of safe edges does not contain a cycle.

Proof.
Assume false, and let $\pi$ a cycle made of safe edges.

- $e$: Most expensive edge in the cycle $\pi$.
- $\mathcal{C} = (S, V \setminus S)$: Cut that $e$ is safe for.
- $\pi$ must have at least two edges in $\mathcal{C}$.

- $f$: cheapest edge in $\pi \cap \mathcal{C}$.
- $e$ is not cheapest edge in $\mathcal{C}$.
A contradiction.
Lemma 20.4.

Let $G$ be a connected graph with distinct edge costs, then the set of safe edges does not contain a cycle.

Proof.

Assume false, and let $\pi$ a cycle made of safe edges.
- $e$: Most expensive edge in the cycle $\pi$.
- $C = (S, V \setminus S)$: Cut that $e$ is safe for.
- $\pi$ must have at least two edges in $C$.
- $f$: Cheapest edge in $\pi \cap C$.
- $e$ is not cheapest edge in $C$.

A contradiction.
Safe Edges form an MST

**Corollary 20.5.**

Let $G$ be a connected graph with distinct edge costs, then set of safe edges form the unique MST of $G$.

**Consequence:** Every correct MST algorithm when $G$ has unique edge costs includes exactly the safe edges.
Corollary 20.5.

Let $G$ be a connected graph with distinct edge costs, then set of safe edges form the unique MST of $G$.

Consequence: Every correct MST algorithm when $G$ has unique edge costs includes exactly the safe edges.
THE END

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(for now)