19.6.2

Interval Scheduling: Earliest finish time
Earliest Finish Time

Process jobs in the order of their finishing times, beginning with those that finish earliest.
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Optimal Greedy Algorithm

\[ R \text{ is the set of all requests} \]
\[ X \leftarrow \emptyset \hspace{1em} (* X \text{ stores the jobs that will be scheduled } *) \]
\[ \textbf{while } R \text{ is not empty} \]
\[ \hspace{1em} \text{choose } i \in R \text{ such that finishing time of } i \text{ is smallest} \]
\[ \hspace{1em} \text{add } i \text{ to } X \]
\[ \hspace{1em} \text{remove from } R \text{ all requests that overlap with } i \]
\[ \textbf{return } X \]

**Theorem 19.2.**

The greedy algorithm that picks jobs in the order of their finishing times is optimal.
Implementation and Running Time

Initially $R$ is the set of all requests
$X \leftarrow \emptyset$ (* $X$ stores the jobs that will be scheduled *)
while $R$ is not empty
    choose $i \in R$ such that finishing time of $i$ is least
    if $i$ does not overlap with requests in $X$
        add $i$ to $X$
    remove $i$ from $R$
return the set $X$

- Presort all requests based on finishing time. $O(n \log n)$ time
- Now choosing least finishing time is $O(1)$
- Keep track of the finishing time of the last request added to $A$. Then check if starting time of $i$ later than that
- Thus, checking non-overlapping is $O(1)$
- Total time $O(n \log n + n) = O(n \log n)$
Comments

Interesting Exercise: smallest interval first picks at least half the optimum number of intervals.

All requests need not be known at the beginning. Such **online** algorithms are a subject of research.
Weighted Interval Scheduling

Suppose we are given $n$ jobs. Each job $i$ has a start time $s_i$, a finish time $f_i$, and a weight $w_i$. We would like to find a set $S$ of compatible jobs whose total weight is maximized. Which of the following greedy algorithms finds the optimum schedule?

- Earliest start time first.
- Earliest finish time first.
- Highest weight first.
- None of the above.
- IDK.

Weighted problem can be solved via dynamic programming. See notes.
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Weighted problem can be solved via dynamic programming. See notes.
THE END

...(for now)