18.1.2
But wait! Things get worse: Negative cycles
Definition 18.2.

A cycle $C$ is a negative length cycle if the sum of the edge lengths of $C$ is negative.
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Negative Length Cycles

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A cycle \( C \) is a negative length cycle if the sum of the edge lengths of \( C \) is negative.

What is the shortest path distance between \( s \) and \( t \)?
Reminder: Paths have to be simple...
Shortest Paths and Negative Cycles

Given $G = (V, E)$ with edge lengths and $s, t$. Suppose

1. $G$ has a negative length cycle $C$, and
2. $s$ can reach $C$ and $C$ can reach $t$.

**Question:** What is the shortest distance from $s$ to $t$?

**Possible answers:** Define shortest distance to be:

1. undefined, that is $-\infty$, OR
2. the length of a shortest simple path from $s$ to $t$. 
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Really bad new about negative edges, and shortest path...

**Lemma 18.3.**

If there is an efficient algorithm to find a shortest simple \( s \rightarrow t \) path in a graph with negative edge lengths, then there is an efficient algorithm to find the longest simple \( s \rightarrow t \) path in a graph with positive edge lengths.

Finding the \( s \rightarrow t \) longest path is difficult. **NP-Hard!**
THE END

... (for now)