17.4 Shortest path trees and variants
17.4.1
Shortest Path Tree
Dijkstra’s algorithm finds the shortest path distances from \( s \) to \( V \).

**Question:** How do we find the paths themselves?

```plaintext
Q = makePQ()
insert(Q, (s, 0))
prev(s) ← null
for each node \( u \neq s \) do
    insert(Q, (u, ∞))
    prev(u) ← null
X = ∅
for \( i = 1 \) to \( |V| \) do
    \((v, \text{dist}(s, v)) = \text{extractMin}(Q)\)
    \(X = X \cup \{v\}\)
    for each \( u \) in \( \text{Adj}(v) \) do
        if \( \text{dist}(s, v) + \ell(v, u) < \text{dist}(s, u) \) then
            decreaseKey(Q, (u, \text{dist}(s, v) + \ell(v, u)))
            prev(u) = v
```
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  \(X = X \cup \{v\}\)
  for each \( u \) in Adj(v) do
    if \((dist(s, v) + ℓ(v, u) < dist(s, u))\) then
      decreaseKey(Q, (u, dist(s, v) + ℓ(v, u)))
      prev(u) = v
```
Lemma

The edge set \((u, \text{prev}(u))\) is the reverse of a shortest path tree rooted at \(s\). For each \(u\), the reverse of the path from \(u\) to \(s\) in the tree is a shortest path from \(s\) to \(u\).

Proof Sketch.

1. The edge set \(\{(u, \text{prev}(u)) \mid u \in V\}\) induces a directed in-tree rooted at \(s\) (Why?)
2. Use induction on \(|X|\) to argue that the tree is a shortest path tree for nodes in \(V\).
Shortest paths to $s$

Dijkstra’s algorithm gives shortest paths from $s$ to all nodes in $V$. How do we find shortest paths from all of $V$ to $s$?

1. In undirected graphs shortest path from $s$ to $u$ is a shortest path from $u$ to $s$ so there is no need to distinguish.

2. In directed graphs, use Dijkstra’s algorithm in $G^{\text{rev}}$. 
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2. In directed graphs, use Dijkstra’s algorithm in $G^{rev}$!
THE END

... (for now)