16.6.3
The linear-time SCC algorithm itself

SCC
Linear Time Algorithm

...for computing the strong connected components in $G$

```latex
\begin{algorithm}
\textbf{do} $\text{DFS}(G^\text{rev})$ and output vertices in decreasing post order.
Mark all nodes as unvisited
\textbf{for} each $u$ in the computed order \textbf{do}
    \textbf{if} $u$ is not visited \textbf{then}
        $\text{DFS}(u)$
        Let $S_u$ be the nodes reached by $u$
        Output $S_u$ as a strong connected component
        Remove $S_u$ from $G$
\end{algorithm}
```

Theorem

Algorithm runs in time $O(m + n)$ and correctly outputs all the SCCs of $G$. 
Linear Time Algorithm: An Example - Initial steps 1

Graph $G$:

Reverse graph $G^{rev}$:

DFS of reverse graph:
Reverse graph $G^{\text{rev}}$:

DFS of reverse graph:

Pre/Post DFS numbering of reverse graph:
Linear Time Algorithm: An Example

Removing connected components: 1

Original graph $G$ with rev post numbers:

Do **DFS** from vertex $G$ remove it.

$\text{SCC computed: } \{G\}$
Linear Time Algorithm: An Example

Removing connected components: 2

Do **DFS** from vertex **G**
remove it.

SCC computed:
\{G\}

Do **DFS** from vertex **H**, remove it.

SCC computed:
\{G\}, \{H\}
Linear Time Algorithm: An Example

Removing connected components: 3

Do **DFS** from vertex \( H \), remove it.

\[
\begin{array}{c}
B & 12 \\
& \leftarrow\\
A & 6 \\
& \leftarrow\\
C & 4 \\
D & 5 \\
E & 11 \\
& \leftarrow\\
F & 10 \\
& \leftarrow\\
\end{array}
\]

**SCC** computed: \( \{G\}, \{H\} \)

Do **DFS** from vertex \( B \).
Remove visited vertices: \( \{F, B, E\} \).

\[
\begin{array}{c}
A & 6 \\
& \leftarrow\\
C & 4 \\
D & 5 \\
\end{array}
\]

**SCC** computed: \( \{G\}, \{H\}, \{F, B, E\} \)
Linear Time Algorithm: An Example

Removing connected components: 4

Do **DFS** from vertex **F**

Remove visited vertices: **\{F, B, E\}**.

SCC computed: **\{G\}, \{H\}, \{F, B, E\}**

\[ 
\begin{array}{c}
A \rightarrow C \\
C \rightarrow D \\
D \rightarrow A
\end{array} \]

Do **DFS** from vertex **A**

Remove visited vertices: **\{A, C, D\}**.

SCC computed: **\{G\}, \{H\}, \{F, B, E\}, \{A, C, D\}**
Linear Time Algorithm: An Example

Final result

SCC computed:
{G}, {H}, {F, B, E}, {A, C, D}

Which is the correct answer!
Obtaining the meta-graph...
Once the strong connected components are computed.

Exercise:
Given all the strong connected components of a directed graph $G = (V, E)$ show that the meta-graph $G^{SCC}$ can be obtained in $O(m + n)$ time.
Solving Problems on Directed Graphs

A template for a class of problems on directed graphs:

- Is the problem solvable when $G$ is strongly connected?
- Is the problem solvable when $G$ is a DAG?
- If the above two are feasible then is the problem solvable in a general directed graph $G$ by considering the meta graph $G^{SCC}$?
THE END

... 

(for now)