16.6.3

The linear-time SCC algorithm itself

SCC

Linear Time Algorithm

...for computing the strong connected components in G

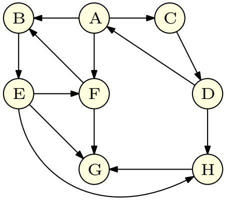
```
do DFS(G^rev) and output vertices in decreasing post order.
Mark all nodes as unvisited
for each u in the computed order do
    if u is not visited then
        DFS(u)
        Let S_u be the nodes reached by u
        Output S_u as a strong connected component
        Remove S_u from G
```

Theorem

Algorithm runs in time $O(m + n)$ and correctly outputs all the SCCs of G.

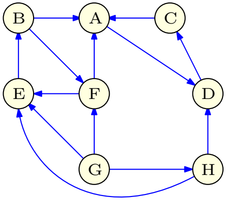
Linear Time Algorithm: An Example - Initial steps 1

Graph G:



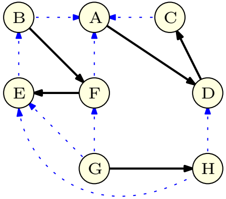
$\Longrightarrow$

Reverse graph G^{rev}:



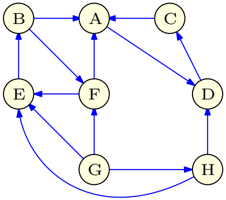
$\Longrightarrow$

DFS of reverse graph:



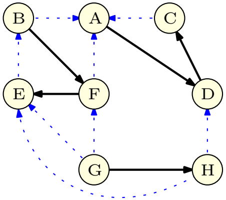
Linear Time Algorithm: An Example - Initial steps 2

Reverse graph $G^{\mathbf{rev}}$:



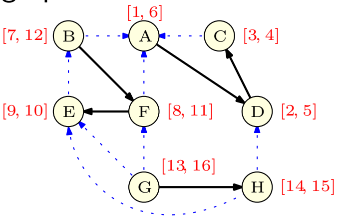
$\Longrightarrow$

DFS of reverse graph:



$\Longrightarrow$

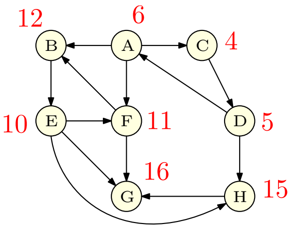
Pre/Post **DFS** numbering of reverse graph:



Linear Time Algorithm: An Example

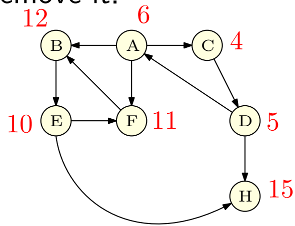
Removing connected components: 1

Original graph G with rev post numbers:



$\Longrightarrow$

Do **DFS** from vertex G remove it.



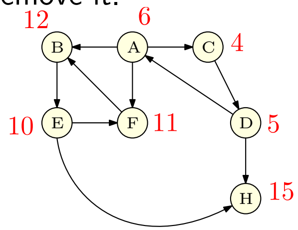
SCC computed:

$\{G\}$

Linear Time Algorithm: An Example

Removing connected components: 2

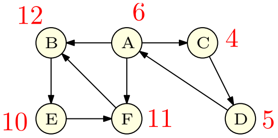
Do **DFS** from vertex G remove it.



SCC computed:
$\{G\}$

$\Longrightarrow$

Do **DFS** from vertex H, remove it.

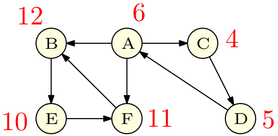


SCC computed:
$\{G\}, \{H\}$

Linear Time Algorithm: An Example

Removing connected components: 3

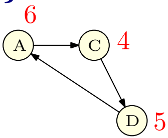
Do **DFS** from vertex H, remove it.



SCC computed:
$\{G\}, \{H\}$

$\Longrightarrow$

Do **DFS** from vertex B
Remove visited vertices:
$\{F, B, E\}$.



SCC computed:
$\{G\}, \{H\}, \{F, B, E\}$

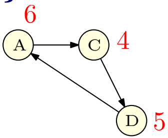
Linear Time Algorithm: An Example

Removing connected components: 4

Do **DFS** from vertex F

Remove visited vertices:

$\{F, B, E\}$.



SCC computed:

$\{G\}, \{H\}, \{F, B, E\}$

$\Longrightarrow$

Do **DFS** from vertex A

Remove visited vertices:

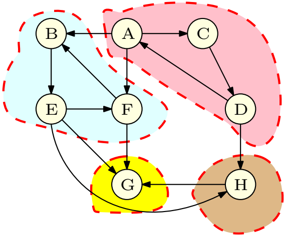
$\{A, C, D\}$.

SCC computed:

$\{G\}, \{H\}, \{F, B, E\}, \{A, C, D\}$

Linear Time Algorithm: An Example

Final result



SCC computed:

$\{G\}, \{H\}, \{F, B, E\}, \{A, C, D\}$

Which is the correct answer!

Obtaining the meta-graph...

Once the strong connected components are computed.

Exercise:

Given all the strong connected components of a directed graph $G = (V, E)$ show that the meta-graph G^{SCC} can be obtained in $O(m + n)$ time.

Solving Problems on Directed Graphs

A template for a class of problems on directed graphs:

- Is the problem solvable when G is strongly connected?
- Is the problem solvable when G is a DAG?
- If the above two are feasible then is the problem solvable in a general directed graph G by considering the meta graph G^{SCC}?

THE END

...

(for now)