16.5
The meta graph of strong connected components
Strong Connected Components (SCCs)

Algorithmic Problem
Find all SCCs of a given directed graph.

Previous lecture:
Saw an $O(n \cdot (n + m))$ time algorithm.
This lecture: sketch of a $O(n + m)$ time algorithm.
Graph of SCCs

\[ G: \]

Meta-graph of SCCs

Let \( S_1, S_2, \ldots, S_k \) be the strong connected components (i.e., SCCs) of \( G \). The graph of SCCs is \( G^{SCC} \)

1. Vertices are \( S_1, S_2, \ldots, S_k \)
2. There is an edge \((S_i, S_j)\) if there is some \( u \in S_i \) and \( v \in S_j \) such that \((u, v)\) is an edge in \( G \).
Reversal and SCCs

**Proposition**

For any graph $G$, the graph of SCCs of $G^{\text{rev}}$ is the same as the reversal of $G^{\text{SCC}}$.

**Proof.**

Exercise.
The meta graph of SCCs is a DAG...

**Proposition**

*For any graph* $G$, the graph $G^{\text{SCC}}$ *has no directed cycle.*

**Proof.**

If $G^{\text{SCC}}$ has a cycle $S_1, S_2, \ldots, S_k$ then $S_1 \cup S_2 \cup \cdots \cup S_k$ should be in the same SCC in $G$. Formal details: exercise.
To Remember: Structure of Graphs

**Undirected graph:** connected components of $G = (V, E)$ partition $V$ and can be computed in $O(m + n)$ time.

**Directed graph:** the meta-graph $G^{SCC}$ of $G$ can be computed in $O(m + n)$ time. $G^{SCC}$ gives information on the partition of $V$ into strong connected components and how they form a DAG structure.

Above structural decomposition will be useful in several algorithms.
THE END

... (for now)