16.2
Directed Acyclic Graphs

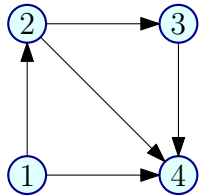
16.2.1
DAGs definition and basic properties

DAG

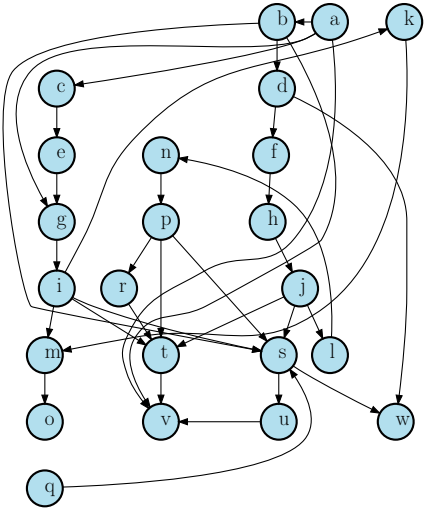
Directed Acyclic Graphs

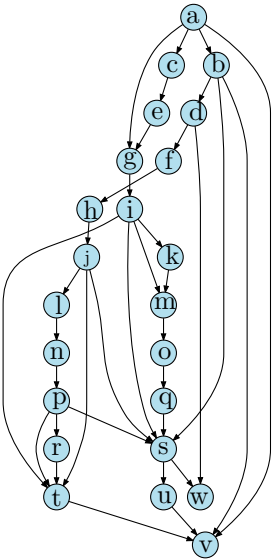
Definition

A directed graph G is a **directed acyclic graph** (DAG) if there is no directed cycle in G.

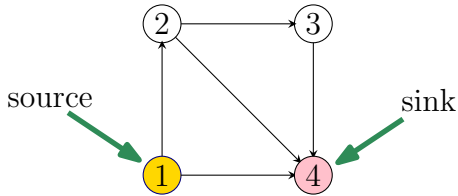


Is this a DAG?





Sources and Sinks



Definition

1. A vertex ***u*** is a **source** if it has no in-coming edges.
2. A vertex ***u*** is a **sink** if it has no out-going edges.

Simple DAG Properties

Proposition

Every DAG G has at least one source and at least one sink.

Proof.

Let $P = v_1, v_2, \ldots, v_k$ be a longest path in G. Claim that v_1 is a source and v_k is a sink. Suppose not. Then v_1 has an incoming edge which either creates a cycle or a longer path both of which are contradictions. Similarly if v_k has an outgoing edge. □

Simple DAG Properties

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DAG properties

1. G is a DAG if and only if G^{rev} is a DAG.
2. G is a DAG if and only each node is in its own strong connected component.

Formal proofs: exercise.

THE END

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(for now)