15.4.1

Strong connected components
Connectivity and Strong Connected Components

**Definition**

Given a directed graph $G$, $u$ is strongly connected to $v$ if $u$ can reach $v$ and $v$ can reach $u$. In other words $v \in \text{rch}(u)$ and $u \in \text{rch}(v)$.

Define relation $C$ where $uCv$ if $u$ is (strongly) connected to $v$.

**Proposition**

$C$ is an equivalence relation, that is reflexive, symmetric and transitive.

Equivalence classes of $C$: strong connected components of $G$.
They partition the vertices of $G$.

$\text{SCC}(u)$: strongly connected component containing $u$. 


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Strongly Connected Components: Example
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Directed Graph Connectivity Problems

1. Given $G$ and nodes $u$ and $v$, can $u$ reach $v$?
2. Given $G$ and $u$, compute $rch(u)$.
3. Given $G$ and $u$, compute all $v$ that can reach $u$, that is all $v$ such that $u \in rch(v)$.
4. Find the strongly connected component containing node $u$, that is $SCC(u)$.
5. Is $G$ strongly connected (a single strong component)?
6. Compute all strongly connected components of $G$. 
THE END

... (for now)