12.4.1
Running time analysis
Running time of LIS([1..n])

\begin{align*}
\text{LIS}_\text{smaller}(A[1..n], x): \\
&\text{if } (n = 0) \text{ then return 0} \\
&m = \text{LIS}_\text{smaller}(A[1..(n - 1)], x) \\
&\text{if } (A[n] < x) \text{ then} \\
&m = \max(m, 1 + \text{LIS}_\text{smaller}(A[1..(n - 1)], A[n])) \\
\text{Output } m
\end{align*}

\begin{align*}
\text{LIS}(A[1..n]): \\
&\text{return } \text{LIS}_\text{smaller}(A[1..n], \infty)
\end{align*}
Lemma

\textbf{LIS\_smaller} \textit{runs in } \textbf{O}(2^n) \textit{ time.}

Improvement: From \textbf{O}(n2^n) to \textbf{O}(2^n).

....one can do much better using memoization!
Running time of LIS([1..n])

Lemma

**LIS**smaller runs in $O(2^n)$ time.

Improvement: From $O(n2^n)$ to $O(2^n)$.

....one can do much better using memoization!
Running time of LIS([1..n])

Lemma

LIS\_smaller runs in $O(2^n)$ time.

Improvement: From $O(n2^n)$ to $O(2^n)$.

....one can do much better using memoization!
THE END

...(for now)