11.4.2
Quick select
QuickSelect
Divide and Conquer Approach

1. Pick a pivot element $a$ from $A$
2. Partition $A$ based on $a$.
   $A_{\text{less}} = \{x \in A \mid x \leq a\}$ and $A_{\text{greater}} = \{x \in A \mid x > a\}$
3. $|A_{\text{less}}| = j$: return $a$
4. $|A_{\text{less}}| > j$: recursively find $j$th smallest element in $A_{\text{less}}$
5. $|A_{\text{less}}| < j$: recursively find $k$th smallest element in $A_{\text{greater}}$ where $k = j - |A_{\text{less}}|$.
Example

| 16 | 14 | 34 | 20 | 12 | 5 | 3 | 19 | 11 |
Time Analysis

1. Partitioning step: $O(n)$ time to scan $A$

2. How do we choose pivot? Recursive running time?

Suppose we always choose pivot to be $A[1]$.

Say $A$ is sorted in increasing order and $j = n$.
Exercise: show that algorithm takes $\Omega(n^2)$ time.
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A Better Pivot

Suppose pivot is the $\ell$th smallest element where $n/4 \leq \ell \leq 3n/4$. That is pivot is approximately in the middle of $A$

Then $n/4 \leq |A_{\text{less}}| \leq 3n/4$ and $n/4 \leq |A_{\text{greater}}| \leq 3n/4$. If we apply recursion,

$$T(n) \leq T(3n/4) + O(n)$$

Implies $T(n) = O(n)$!

How do we find such a pivot? Randomly? In fact works!
Analysis a little bit later.

Can we choose pivot deterministically?
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THE END

... (for now)