10.4
Recursion as self reductions
Recursion

Reduction: reduce one problem to another

Recursion: a special case of reduction
1. reduce problem to a smaller instance of itself
2. self-reduction

1. Problem instance of size $n$ is reduced to one or more instances of size $n - 1$ or less.
2. For termination, problem instances of small size are solved by some other method as base cases.
Recursion

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Recursion

1. Recursion is a very powerful and fundamental technique
2. Basis for several other methods
   1. Divide and conquer
   2. Dynamic programming
   3. Enumeration and branch and bound etc
   4. Some classes of greedy algorithms
3. Makes proof of correctness easy (via induction)
4. Recurrences arise in analysis
Move stack of $n$ disks from peg $0$ to peg $2$, one disk at a time. 

**Rule:** cannot put a larger disk on a smaller disk.

**Question:** what is a strategy and how many moves does it take?
Tower of Hanoi via Recursion

The Tower of Hanoi algorithm; ignore everything but the bottom disk
Recursive Algorithm

\[
\text{Hanoi}(n, \text{src}, \text{dest}, \text{tmp}):
\]
\[
\text{if } (n > 0) \text{ then}
\]
\[
\text{Hanoi}(n - 1, \text{src}, \text{tmp}, \text{dest})
\]
\[
\text{Move disk } n \text{ from src to dest}
\]
\[
\text{Hanoi}(n - 1, \text{tmp}, \text{dest}, \text{src})
\]

\(T(n)\): time to move \(n\) disks via recursive strategy

\[
T(n) = 2T(n - 1) + 1 \quad n > 1 \quad \text{and} \quad T(1) = 1
\]
Recursive Algorithm

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\text{Hanoi}(n, \text{src}, \text{dest}, \text{tmp}): \\
\quad \text{if } (n > 0) \text{ then} \\
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\begin{align*}
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& \text{Move disk } n \text{ from src to dest} \\
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\]
\[ T(n) = 2T(n - 1) + 1 \]
\[ = 2^2T(n - 2) + 2 + 1 \]
\[ = \ldots \]
\[ = 2^iT(n - i) + 2^{i-1} + 2^{i-2} + \ldots + 1 \]
\[ = \ldots \]
\[ = 2^{n-1}T(1) + 2^{n-2} + \ldots + 1 \]
\[ = 2^{n-1} + 2^{n-2} + \ldots + 1 \]
\[ = (2^n - 1)/(2 - 1) = 2^n - 1 \]
THE END

...  

(for now)