8.3 Snapshots, computation as sequence of strings
Snapshots = ID: Instantaneous Description

1. Contains all necessary information to capture “state of the computation”.
2. Includes
   1. state $q$ of $M$
   2. location of read/write head
   3. contents of tape from left edge to rightmost non-blank (or to head, whichever is rightmost).
As a string

\[ x_1 x_2 \cdots x_i \cdots x_n \]

ID: \( x_1 x_2 \cdots x_{i-1} q x_i x_{i+1} \cdots x_n \)

\( x_1, \ldots, x_n \in \Gamma, \ q \in Q. \)
A step in computation as rewriting strings

\[ x_1 x_2 \ldots x_{i-1} q x_i x_{i+1} \ldots x_n \]

If transition is \( \delta(q, X_i) = (p, Y, \Lambda) \), new ID is:

\[
\text{current ID : } \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad x_1 x_2 \ldots x_{i-2} x_{i-1} q x_i x_{i+1} \ldots x_n
\]

\[
\delta(q, X_i) = (p, y, \Lambda) \implies x_1 x_2 \ldots x_{i-2} p x_{i-1} y x_{i+1} \ldots x_n
\]

If no transition defined, or illegal transition, then no next ID (crash).

**Shockingly:** Computation is just a string rewriting system.
A step in computation as rewriting strings

\[ x_1 x_2 \ldots x_{i-1} q x_i x_{i+1} \ldots x_n \]

If transition is \( \delta(q, X_i) = (p, Y, L) \), new ID is:

\[
\text{current ID : } x_1 x_2 \ldots x_{i-2} x_{i-1} q x_i x_{i+1} \ldots x_n \\
\delta(q, X_i) = (p, y, L) \implies x_1 x_2 \ldots x_{i-2} p x_{i-1} y x_{i+1} \ldots x_n
\]

If no transition defined, or illegal transition, then no next ID (crash).

**Shockingly:** Computation is just a string rewriting system.
A step in computation as rewriting strings

1. Initial ID: $q_0w$:
2. Accepting ID: $\alpha q_{\text{acc}} \alpha'$, for some $\alpha, \alpha' \in \Gamma^*$.
3. Rejecting ID: $\alpha q_{\text{rej}} \alpha'$, for some $\alpha, \alpha' \in \Gamma^*$.
4. $I \rightsquigarrow J$: Denotes that if we start execution of TM with configuration/ID encoded by $I$, leads TM (after maybe several steps) to ID $J$.
5. $M$ accepts $w$: If for some $\alpha, \alpha' \in \Gamma^*$, we have $q_0 w \rightsquigarrow \alpha q_{\text{acc}} \alpha'$.

Acceptance happens as soon as TM enters accept state.

6. Language of TM $M$: $L(M) = \{ w \in \Sigma^* | M \text{ accepts } w \}$. 

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A step in computation as rewriting strings

1. Initial ID: $q_0w$:
2. Accepting ID: $\alpha q_{\text{acc}} \alpha'$, for some $\alpha, \alpha' \in \Gamma^*$.
3. Rejecting ID: $\alpha q_{\text{rej}} \alpha'$, for some $\alpha, \alpha' \in \Gamma^*$.
4. $\mathcal{I} \leadsto \mathcal{J}$: Denotes that if we start execution of $\text{TM}$ with configuration/ID encoded by $\mathcal{I}$, leads $\text{TM}$ (after maybe several steps) to ID $\mathcal{J}$
5. $M$ accepts $w$: If for some $\alpha, \alpha' \in \Gamma^*$, we have
   \[
   q_0w \leadsto \alpha q_{\text{acc}} \alpha'.
   \]
   Acceptance happens as soon as $\text{TM}$ enters accept state.
6. Language of $\text{TM} M$: $L(M) = \{w \in \Sigma^* \mid M$ accepts $w\}$. 

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Acceptance happens as soon as $\text{TM}$ enters accept state.

6. Language of $\text{TM} M$: $L(M) = \{w \in \Sigma^* \mid M \text{ accepts } w\}$. 
Non-accepting computation

*M does not accept* $w$ if:

1. *$M$ enters* $q_{\text{rej}}$ (i.e., *$M$ rejects* $w$)
2. *$M$ crashes* (moves to left of tape, no transition available, etc).
3. *$M$ runs forever*.

If the *TM* keeps running, should we wait, or is it rejection?
Non-accepting computation

$M$ does not accept $w$ if:

1. $M$ enters $q_{\text{rej}}$ (i.e., $M$ rejects $w$)
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3. $M$ runs forever.

If the TM keeps running, should we wait, or is it rejection?
Everything is a number
THE END

...(for now)