7.4.2
Parse trees and ambiguity
Parse Trees or Derivation Trees

A tree to represent the derivation $S \Rightarrow^* w$.

- Rooted tree with root labeled $S$
- Non-terminals at each internal node of tree
- Terminals at leaves
- Children of internal node indicate how non-terminal was expanded using a production rule

A picture is worth a thousand words
Parse Trees or Derivation Trees

A tree to represent the derivation $S \leadsto^* w$.

- Rooted tree with root labeled $S$
- Non-terminals at each internal node of tree
- Terminals at leaves
- Children of internal node indicate how non-terminal was expanded using a production rule

A picture is worth a thousand words
Example

A derivation tree for abbaab
(also called “parse tree”)

A corresponding derivation of abbaab

\[
S \rightarrow aSb \mid bSa \mid SS \mid ab \mid ba \mid \epsilon
\]
Ambiguity in CFLs

Definition
A CFG $G$ is ambiguous if there is a string $w \in L(G)$ with two different parse trees. If there is no such string then $G$ is unambiguous.

Example: $S \rightarrow S - S \mid 1 \mid 2 \mid 3$
Ambiguity in CFLs

- Original grammar: \( S \rightarrow S - S \mid 1 \mid 2 \mid 3 \)
- Unambiguous grammar:
  \[
  S \rightarrow S - C \mid 1 \mid 2 \mid 3 \\
  C \rightarrow 1 \mid 2 \mid 3
  \]

The grammar forces a parse corresponding to left-to-right evaluation.
Inherently ambiguous languages

**Definition**

A **CFL** $L$ is inherently ambiguous if there is no unambiguous **CFG** $G$ such that $L = L(G)$.

- There exist inherently ambiguous CFLs.
  
  **Example:** $L = \{a^n b^m c^k | n = m \text{ or } m = k\}$

- Given a grammar $G$ it is **undecidable** to check whether $L(G)$ is inherently ambiguous. No algorithm!
Inherently ambiguous languages

Definition

A CFL $L$ is inherently ambiguous if there is no unambiguous CFG $G$ such that $L = L(G)$.

- There exist inherently ambiguous CFLs.
  - Example: $L = \{a^n b^m c^k \mid n = m \text{ or } m = k\}$
- Given a grammar $G$ it is undecidable to check whether $L(G)$ is inherently ambiguous. No algorithm!
Inherently ambiguous languages

**Definition**

A CFL \( L \) is inherently ambiguous if there is no unambiguous CFG \( G \) such that \( L = L(G) \).

- There exist inherently ambiguous CFLs.
  - **Example**: \( L = \{ a^n b^m c^k \mid n = m \text{ or } m = k \} \)
- Given a grammar \( G \) it is **undecidable** to check whether \( L(G) \) is inherently ambiguous. No algorithm!
THE END

... 

(for now)