6.4

Closure properties: Proving non-regularity
Non-regularity via closure properties

\[ H = \{ \text{bitstrings with equal number of 0s and 1s} \} \]

\[ H' = \{ 0^k 1^k \mid k \geq 0 \} \]

Suppose we have already shown that \( L' \) is non-regular. Can we show that \( L \) is non-regular without using the fooling set argument from scratch?

\[ H' = H \cap L(0^*1^*) \]

Claim: The above and the fact that \( L' \) is non-regular implies \( L \) is non-regular. Why?

Suppose \( H \) is regular. Then since \( L(0^*1^*) \) is regular, and regular languages are closed under intersection, \( H' \) also would be regular. But we know \( H' \) is not regular, a contradiction.
Non-regularity via closure properties

\[ H = \{ \text{bitstrings with equal number of 0s and 1s} \} \]

\[ H' = \{ 0^k1^k \mid k \geq 0 \} \]

Suppose we have already shown that \( L' \) is non-regular. Can we show that \( L \) is non-regular without using the fooling set argument from scratch?

\[ H' = H \cap L(0^*1^*) \]

Claim: The above and the fact that \( L' \) is non-regular implies \( L \) is non-regular. Why?

Suppose \( H \) is regular. Then since \( L(0^*1^*) \) is regular, and regular languages are closed under intersection, \( H' \) also would be regular. But we know \( H' \) is not regular, a contradiction.
Non-regularity via closure properties

\[ H = \{ \text{bitstrings with equal number of 0s and 1s} \} \]

\[ H' = \{ 0^k1^k \mid k \geq 0 \} \]

Suppose we have already shown that \( L' \) is non-regular. Can we show that \( L \) is non-regular without using the fooling set argument from scratch?

\[ H' = H \cap L(0^*1^*) \]

**Claim:** The above and the fact that \( L' \) is non-regular implies \( L \) is non-regular. Why?

Suppose \( H \) is regular. Then since \( L(0^*1^*) \) is regular, and regular languages are closed under intersection, \( H' \) also would be regular. But we know \( H' \) is not regular, a contradiction.
Non-regularity via closure properties

General recipe:

Apply closure properties

$L_1$

$L_2$

$L_n$

$L_?$

$L_{\text{non-regular}}$
Proving non-regularity: Summary

- Method of distinguishing suffixes. To prove that $L$ is non-regular find an infinite fooling set.
- Closure properties. Use existing non-regular languages and regular languages to prove that some new language is non-regular.
- **Pumping lemma.** We did not cover it but it is sometimes an easier proof technique to apply, but not as general as the fooling set technique.
THE END

... (for now)