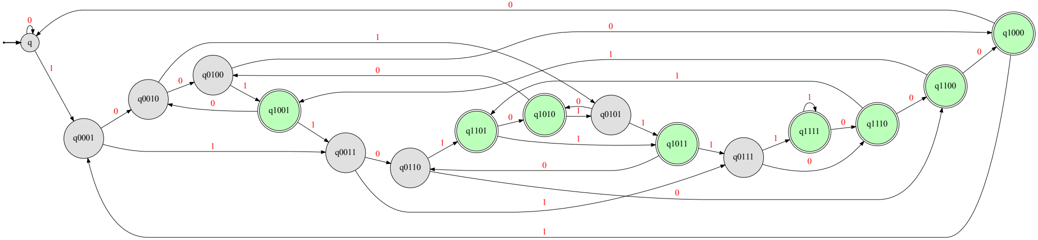
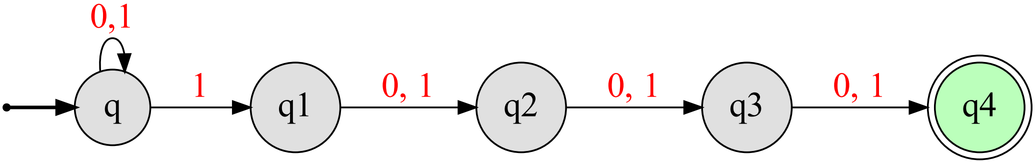
6.3.1

Exponential gap in number of states between DFA and NFA sizes

$L_4 = \{w \in \{0,1\}^* \mid w \text{ has a 1 located 4 positions from the end}\}$



DFA:



NFA:

Exponential gap between NFA and DFA size

$L_k = \{w \in \{0,1\}^* \mid w \text{ has a 1 } k \text{ positions from the end}\}$

Recall that L_k is accepted by a NFA N with $k+1$ states.

Theorem

Every DFA that accepts L_k has at least 2^k states.

Claim

$F = \{w \in \{0,1\}^* : |w| = k\}$ is a fooling set of size 2^k for L_k.

Why?

- Suppose $a_1 a_2 \ldots a_k$ and $b_1 b_2 \ldots b_k$ are two distinct bitstrings of length k
- Let i be first index where $a_i \neq b_i$
- $y = 0^{k-i-1}$ is a distinguishing suffix for the two strings

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How do pick a fooling set

How do we pick a fooling set F?

- If x, y are in F and $x \neq y$ they should be distinguishable! Of course.
- All strings in F except maybe one should be prefixes of strings in the language L. For example if $L = \{0^k 1^k \mid k \geq 0\}$ do not pick 1 and 10 (say). Why?

THE END

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(for now)