5.2
Closure Properties of Regular Languages
Regular Languages

Regular languages have three different characterizations

- Inductive definition via base cases and closure under union, concatenation and Kleene star
- Languages accepted by **DFA**s
- Languages accepted by **NFA**s

Regular language closed under many operations:

- union, concatenation, Kleene star via inductive definition or **NFA**s
- complement, union, intersection via **DFA**s
- homomorphism, inverse homomorphism, reverse, ...

Different representations allow for flexibility in proofs
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Different representations allow for flexibility in proofs
Example: PREFIX

Let $L$ be a language over $\Sigma$.

**Definition**

$\text{PREFIX}(L) = \{w \mid wx \in L, x \in \Sigma^*\}$

**Theorem**

If $L$ is regular then $\text{PREFIX}(L)$ is regular.

Let $M = (Q, \Sigma, \delta, s, A)$ be a DFA that recognizes $L$.

$X = \{q \in Q \mid s \text{ can reach } q \text{ in } M\}$  
$Y = \{q \in Q \mid q \text{ can reach some state in } A\}$

$Z = X \cap Y$

Create new DFA $M' = (Q, \Sigma, \delta, s, Z)$

Claim: $L(M') = \text{PREFIX}(L)$. 
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Create new DFA $M' = (Q, \Sigma, \delta, s, Z)$

Claim: $L(M') = \text{PREFIX}(L)$. 
Exercise: SUFFIX

Let $L$ be a language over $\Sigma$.

Definition

$\text{SUFFIX}(L) = \{ w \mid xw \in L, x \in \Sigma^* \}$

Prove the following:

Theorem

If $L$ is regular then $\text{PREFIX}(L)$ is regular.
Exercise: SUFFIX

An alternative “proof” using a figure
THE END

... 

(for now)