5.1.3
Proof of correctness of conversion of NFA to DFA
Proof of Correctness

**Theorem**

Let \( N = (Q, \Sigma, s, \delta, A) \) be a NFA and let \( D = (Q', \Sigma, \delta', s', A') \) be a DFA constructed from \( N \) via the subset construction. Then \( L(N) = L(D) \).

**Stronger claim:**

**Lemma**

For every string \( w \), \( \delta_N^*(s, w) = \delta_D^*(s', w) \).

Proof by induction on \( |w| \).
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For every string $w$, $\delta^*_N(s, w) = \delta^*_D(s', w)$.

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Proof continued I

Lemma

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Proof:

Base case: $w = \epsilon$.

$\delta^*_N(s, \epsilon) = \epsilon \text{reach}(s)$.

$\delta^*_D(s', \epsilon) = s' = \epsilon \text{reach}(s)$ by definition of $s'$.
Lemma

For every string $w$, $\delta^*_N(s, w) = \delta^*_D(s', w)$.

Inductive step: $w = xa$ (Note: suffix definition of strings)

$\delta^*_N(s, xa) = \bigcup_{p \in \delta^*_N(s, x)} \delta^*_N(p, a)$ by inductive definition of $\delta^*_N$

$\delta^*_D(s', xa) = \delta_D(\delta^*_D(s, x), a)$ by inductive definition of $\delta^*_D$

By inductive hypothesis: $Y = \delta^*_N(s, x) = \delta^*_D(s, x)$

Thus $\delta^*_N(s, xa) = \bigcup_{p \in Y} \delta^*_N(p, a) = \delta_D(Y, a)$ by definition of $\delta_D$.

Therefore,

$\delta^*_N(s, xa) = \delta_D(Y, a) = \delta_D(\delta^*_D(s, x), a) = \delta^*_M(s', xa)$. which is what we need.
Lemma

For every string \( w \), \( \delta_N^*(s, w) = \delta_D^*(s', w) \).

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(Note: suffix definition of strings)  
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\delta_N^*(s, xa) = \bigcup_{p \in \delta_N^*(s, x)} \delta_N^*(p, a)
\]
by inductive definition of \( \delta_N^* \)

\[
\delta_D^*(s', xa) = \delta_D(\delta_D^*(s, x), a)
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THE END
...
(for now)