5.1.2 Algorithm for converting NFA to DFA
Recall I
Extending the transition function to strings

**Definition**

For NFA $N = (Q, \Sigma, \delta, s, A)$ and $q \in Q$ the $\epsilon$reach($q$) is the set of all states that $q$ can reach using only $\epsilon$-transitions.

**Definition**

Inductive definition of $\delta^* : Q \times \Sigma^* \rightarrow \mathcal{P}(Q)$:

- if $w = \epsilon$, $\delta^*(q, w) = \epsilon$reach($q$)
- if $w = a$ where $a \in \Sigma$: $\delta^*(q, a) = \epsilon$reach($\bigcup_{p \in \epsilon$reach($q$)} \delta(p, a)$)
- if $w = ax$: $\delta^*(q, w) = \epsilon$reach($\bigcup_{p \in \epsilon$reach($q$)} \bigcup_{r \in \delta^*(p, a)} \delta^*(r, x)$)
Recall II
Formal definition of language accepted by $N$

Definition

A string $w$ is accepted by NFA $N$ if $\delta_N^*(s, w) \cap A \neq \emptyset$.

Definition

The language $L(N)$ accepted by a NFA $N = (Q, \Sigma, \delta, s, A)$ is

$$\{ w \in \Sigma^* \mid \delta^*(s, w) \cap A \neq \emptyset \}.$$
Subset Construction

**NFA** $N = (Q, \Sigma, s, \delta, A)$. We create a **DFA** $D = (Q', \Sigma, \delta', s', A')$ as follows:

- $Q' = \mathcal{P}(Q)$
- $s' = \epsilon \text{reach}(s) = \delta^*(s, \epsilon)$
- $A' = \{X \subseteq Q \mid X \cap A \neq \emptyset\}$
- $\delta'(X, a) = \bigcup_{q \in X} \delta^*(q, a)$ for each $X \subseteq Q, a \in \Sigma$. 
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Incremental construction

Only build states reachable from \( s' = \epsilon \text{reach}(s) \) the start state of \( D \)

\[
\delta'(X, a) = \bigcup_{q \in X} \delta^*(q, a).
\]
An optimization: Incremental algorithm

- Build $D$ beginning with start state $s' == \epsilon \text{reach}(s)$
- For each existing state $X \subseteq Q$ consider each $a \in \Sigma$ and calculate the state $U = \delta'(X, a) = \bigcup_{q \in X} \delta^*(q, a)$ and add a transition.

To compute $Z_{q,a} = \delta^*(q, a)$ - set of all states reached from $q$ on character $a$
  - Compute $X_1 = \epsilon \text{reach}(q)$
  - Compute $Y_1 = \bigcup_{p \in X_1} \delta(p, a)$
  - Compute $Z_{q,a} = \epsilon \text{reach}(Y) = \bigcup_{r \in Y_1} \epsilon \text{reach}(r)$

- If $U$ is a new state add it to reachable states that need to be explored.
An optimization: Incremental algorithm

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THE END

... (for now)