3.3 Complement language
**Question:** If $M$ is a DFA, is there a DFA $M'$ such that $L(M') = \Sigma^* \setminus L(M)$? That is, are languages recognized by DFAs closed under complement?
Complement

Example...

Just flip the state of the states!
Complement

**Theorem**

Languages accepted by **DFA**s are closed under complement.

**Proof.**

Let $M = (Q, \Sigma, \delta, s, A)$ such that $L = L(M)$.

Let $M' = (Q, \Sigma, \delta, s, Q \setminus A)$. Claim: $L(M') = \overline{L}$. Why?

$\delta^*_M = \delta^*_{M'}$. Thus, for every string $w$, $\delta^*_M(s, w) = \delta^*_{M'}(s, w)$.

$\delta^*_M(s, w) \in A \Rightarrow \delta^*_{M'}(s, w) \not\in Q \setminus A$.

$\delta^*_M(s, w) \not\in A \Rightarrow \delta^*_{M'}(s, w) \in Q \setminus A$. $\square$
Complement

Theorem
Languages accepted by DFA s are closed under complement.

Proof.
Let \( M = (Q, \Sigma, \delta, s, A) \) such that \( L = L(M) \).
Let \( M' = (Q, \Sigma, \delta, s, Q \setminus A) \). Claim: \( L(M') = \bar{L} \). Why?

\[ \delta^*_M = \delta^*_{M'} \]. Thus, for every string \( w \), \( \delta^*_M(s, w) = \delta^*_{M'}(s, w) \).

\[ \delta^*_M(s, w) \in A \Rightarrow \delta^*_{M'}(s, w) \notin Q \setminus A \]. \( \delta^*_M(s, w) \notin A \Rightarrow \delta^*_{M'}(s, w) \in Q \setminus A \).
Complement

Theorem

Languages accepted by \textbf{DFA}s are closed under complement.

Proof.

Let $M = (Q, \Sigma, \delta, s, A)$ such that $L = L(M)$. Let $M' = (Q, \Sigma, \delta, s, Q \setminus A)$. Claim: $L(M') = \overline{L}$. Why?

$\delta^*_M = \delta^*_{M'}$. Thus, for every string $w$, $\delta^*_M(s, w) = \delta^*_{M'}(s, w)$.

$\delta^*_M(s, w) \in A \Rightarrow \delta^*_{M'}(s, w) \not\in Q \setminus A$. $\delta^*_M(s, w) \not\in A \Rightarrow \delta^*_{M'}(s, w) \in Q \setminus A$. \hfill $\square$
THE END

... (for now)