3.1.3
Extending the transition function to strings
Extending the transition function to strings

Given DFA $M = (Q, \Sigma, \delta, s, A)$, $\delta(q, a)$ is the state that $M$ goes to from $q$ on reading letter $a$

Useful to have notation to specify the unique state that $M$ will reach from $q$ on reading string $w$

Transition function $\delta^* : Q \times \Sigma^* \rightarrow Q$ defined inductively as follows:

- $\delta^*(q, \epsilon) = q$ if $w = \epsilon$
- $\delta^*(q, ax) = \delta^*(\delta(q, a), x)$ if $w = ax$. 
Extending the transition function to strings

Given DFA $M = (Q, \Sigma, \delta, s, A)$, $\delta(q, a)$ is the state that $M$ goes to from $q$ on reading letter $a$.

Useful to have notation to specify the unique state that $M$ will reach from $q$ on reading string $w$.

Transition function $\delta^* : Q \times \Sigma^* \rightarrow Q$ defined inductively as follows:

1. $\delta^*(q, w) = q$ if $w = \epsilon$
2. $\delta^*(q, w) = \delta^*(\delta(q, a), x)$ if $w = ax$. 
Formal definition of language accepted by $M$

**Definition**

The language $L(M)$ accepted by a DFA $M = (Q, \Sigma, \delta, s, A)$ is

$$\{w \in \Sigma^* \mid \delta^*(s, w) \in A\}.$$
What is:

- $\delta^*(q_1, \epsilon)$
- $\delta^*(q_0, 1011)$
- $\delta^*(q_1, 010)$
- $\delta^*(q_4, 10)$
- So what is $L(M)$???????
What is:
- $\delta^*(q_1, \epsilon)$
- $\delta^*(q_0, 1011)$
- $\delta^*(q_1, 010)$
- $\delta^*(q_4, 10)$
- So what is $L(M)$?
What is:

- $\delta^*(q_1, \epsilon)$
- $\delta^*(q_0, 1011)$
- $\delta^*(q_1, 010)$
- $\delta^*(q_4, 10)$
- So what is $L(M)$??????
What is:

- $\delta^*(q_1, \epsilon)$
- $\delta^*(q_0, 1011)$
- $\delta^*(q_1, 010)$
- $\delta^*(q_4, 10)$

So what is $L(M)$??
What is:

- $\delta^*(q_1, \epsilon)$
- $\delta^*(q_0, 1011)$
- $\delta^*(q_1, 010)$
- $\delta^*(q_4, 10)$
- So what is $L(M)$??????
Example continued

What is $L(M)$ if start state is changed to $q_1$?
Example continued

What is $L(M)$ if final/accept states are set to $\{q_2, q_3\}$ instead of $\{q_0\}$?
Example continued

What is $L(M)$ if final/accept states are set to $\{q_2, q_3\}$ instead of $\{q_0\}$?
What is $L(M)$ if final/accept states are set to $\{q_2, q_3\}$ instead of $\{q_0\}$?
Advantages of formal specification

- Necessary for proofs
- Necessary to specify abstractly for class of languages

**Exercise:** Prove by induction that for any two strings $u, v$, any state $q$, 
\[ \delta^*(q, uv) = \delta^*(\delta^*(q, u), v). \]
THE END

...(for now)