Regular Languages and Expressions

Lecture 2
Thursday, August 27, 2020
2.1

Regular Languages
Regular Languages

A class of simple but useful languages.
The set of regular languages over some alphabet $\Sigma$ is defined inductively as:

1. $\emptyset$ is a regular language.
2. $\{\epsilon\}$ is a regular language.
3. $\{a\}$ is a regular language for each $a \in \Sigma$. Interpreting $a$ as string of length 1.
4. If $L_1, L_2$ are regular then $L_1 \cup L_2$ is regular.
5. If $L_1, L_2$ are regular then $L_1L_2$ is regular.
6. If $L$ is regular, then $L^* = \bigcup_{n \geq 0} L^n$ is regular.
   The $^*$ operator name is Kleene star.
7. If $L$ is regular, then so is $\overline{L} = \Sigma^* \setminus L$.

Regular languages are closed under operations of union, concatenation and Kleene star.
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Regular Languages

Have basic operations to build regular languages.

**Important:** Any language generated by a finite sequence of such operations is regular.

**Lemma**

Let $L_1, L_2, \ldots$, be regular languages over alphabet $\Sigma$. Then the language $\bigcup_{i=1}^{\infty} L_i$ is not necessarily regular.
Some simple regular languages

Lemma

If $w$ is a string then $L = \{ w \}$ is regular.

Example: $\{aba\}$ or $\{abbabbab\}$. Why?

Lemma

Every finite language $L$ is regular.

Examples: $L = \{a, abaab, aba\}$. $L = \{w \mid |w| \leq 100\}$. Why?
**Lemma**

*If* \( w \) *is a string then* \( L = \{ w \} \) *is regular.*

**Example:** \( \{aba\} \) or \( \{abbabbab\} \). Why?

**Lemma**

*Every finite language* \( L \) *is regular.*

**Examples:** \( L = \{a, abaab, aba\} \). \( L = \{w \mid |w| \leq 100\} \). Why?
More Examples

- \{w \mid w \text{ is a keyword in Python program}\}
- \{w \mid w \text{ is a valid date of the form mm/dd/yy}\}
- \{w \mid w \text{ describes a valid Roman numeral}\}
  \{I, II, III, IV, V, VI, VII, VIII, IX, X, XI, \ldots\}
- \{w \mid w \text{ contains ”CS374” as a substring}\}.
Review questions

1. $L_1 \subseteq \{0, 1\}^*$ be a finite language. $L_1$ is a set with finite number of strings. T/F?

2. $L_2 = \{0^i \mid i = 0, 1, \ldots, \infty\}$. The language $L_2$ is regular. T/F?

3. $L_3 = \{0^{2i} \mid i = 0, 1, \ldots, \infty\}$. The language $L_3$ is regular. T/F?

4. $L_4 = \{0^{17i} \mid i = 0, 1, \ldots, \infty\}$. The language $L_4$ is regular. T/F?

5. $L_5 = \{0^i \mid i \text{ is not divisible by 17}\}$. $L_5$ is regular. T/F?

6. $L_6 = \{0^i \mid i \text{ is divisible by 2, 3, or 5}\}$. $L_6$ is regular. T/F?

7. $L_7 = \{0^i \mid i \text{ is divisible by 2, 3, and 5}\}$. $L_7$ is regular. T/F?

8. $L_8 = \{0^i \mid i \text{ is divisible by 2, 3, but not 5}\}$. $L_8$ is regular. T/F?

9. $L_9 = \{0^i1^i \mid i \text{ is divisible by 2, 3, but not 5}\}$. $L_9$ is regular. T/F?

10. $L_{10} = \{w \in \{0, 1\}^* \mid w \text{ has at most 374 1s}\}$. $L_{10}$ is regular. T/F?
THE END

... (for now)