HW 3

Due on **Wednesday, September 16, 2020 at 10am CST**

CS/ECE 374: Algorithms & Models of Computation, Fall 2020 Version: **1.01**

Submission instructions as in previous homeworks.

7 (100 PTS.) NFAs

For each of the following languages over $\Sigma = \{3, 7, 4\}$, draw an NFA that accepts them. Your NFA should have a small number of states (at most say 14 states). Provide a brief explanation for your solution.

7.A. (20 PTS.) $\Sigma^* 3 \Sigma^* 7 \Sigma^* 4 \Sigma^*$

7.B. (20 PTS.) All strings in Σ^* that contain the substrings 374 and 473.

7.C. (20 PTS.) All strings in Σ^* that do not contain 374 as a substring.

7.D. (20 PTS.) All strings in Σ^* that contain the substring 374 and an odd number of 7s.

7.E. (20 PTS.) All strings in Σ^* such that every maximal substring of consecutive 7s is even in size.

8 (100 PTS.) DFAs to NFAs

Given a DFA $M = (\Sigma, Q, \delta, s, A)$ that accepts L, construct an NFA $N = (\Sigma, Q', \delta', s', A')$ that accepts the following languages. You can assume $\Sigma = \{0, 1\}$ in **8.A.** and **8.C.**. Provide a brief explanation for your solution.

8.A. (30 PTS.) $\mathrm{DelOnes}(L) := \left\{ 0^{\#_0(w)} \;\middle|\; w \in L \right\}$; i.e., removes all 1s from the strings of L.

8.B. (30 PTS.) $\mathit{ThereAndBack}(L) := \left\{ xy \;\middle|\; x \in L \text{ and } y^R \in L \right\}$

8.C. (40 PTS.) $\mathit{XOR}(L) := \{ z \mid z = \mathtt{XOR}(x, y) \text{ for some } x \in L, y \in L, \text{ such that } |x| = |y| = |z| \}$, where $\mathtt{XOR}(x, y)$ computes the element-wise XOR of x and y (so for each index i, $z_i = x_i \;\mathtt{XOR}\; y_i$).

8.D. (**Not for submission**) Consider, if you must, the language

$$\mathit{Middle}(L) := \{ y \in L \mid xyz \in L \text{ for some } x, z \text{ such that } |x| = |y| = |z| \}.$$

Prove that this language is regular.

9 (100 PTS.) Fooling Sets

Prove that the following languages are not regular by providing a fooling set. You need to provide an infinite set and also prove that it is a valid fooling set for the given language.

9.A. (20 PTS.) $L = \left\{ w w^R w \;\middle|\; w \in \{0, 1\}^* \right\}$.

9.B. (20 PTS.) $L = \left\{ 0^i 1 0^j \;\middle|\; i \text{ is divisible by } j \right\}$.

9.C. (20 PTS.) $L = \left\{ a^i b^j \;\middle|\; i, j \in \mathbb{N}, \text{ and } j = \log_2 i \right\}$.

9.D. (20 PTS.) $L = \left\{ 0^i 0^j \;\middle|\; i, j \in \mathbb{N}, \text{ and } j = \sqrt{i} \right\}$.

9.E. (20 PTS.) $L = \{ w c d^{\#_a(w)} \mid w \in \{a, b\}^* \}$.