7 (100 pts.) NFAs

For each of the following languages over \( \Sigma = \{3, 7, 4\} \), draw an NFA that accepts them. Your NFA should have a small number of states (at most say 14 states). Provide a brief explanation for your solution.

7.A. (20 pts.) \( \Sigma^*3\Sigma^*7\Sigma^*4\Sigma^* \)
7.B. (20 pts.) All strings in \( \Sigma^* \) that contain the substrings 374 and 473.
7.C. (20 pts.) All strings in \( \Sigma^* \) that do not contain 374 as a substring.
7.D. (20 pts.) All strings in \( \Sigma^* \) that contain the substring 374 and an odd number of 7s.
7.E. (20 pts.) All strings in \( \Sigma^* \) such that every maximal substring of consecutive 7s is even in size.

8 (100 pts.) DFAs to NFAs

Given a DFA \( M = (\Sigma, Q, \delta, s, A) \) that accepts \( L \), construct an NFA \( N = (\Sigma', Q', \delta', s', A') \) that accepts the following languages. You can assume \( \Sigma = \{0, 1\} \) in 8.A and 8.C. Provide a brief explanation for your solution.

8.A. (30 pts.) DelOnes(\( L \)) := \( \{0^{|w|} \mid w \in L \} \); i.e., removes all 1s from the strings of \( L \).
8.B. (30 pts.) ThereAndBack(\( L \)) := \( \{xy \mid x \in L \text{ and } y^R \in L \} \)
8.C. (40 pts.) XOR(\( L \)) := \( \{z \mid z = \text{XOR}(x, y) \text{ for some } x \in L, y \in L, \text{ such that } |x| = |y| = |z|\} \), where XOR(\( x, y \)) computes the element-wise XOR of \( x \) and \( y \) (so for each index \( i \), \( z_i = x_i \text{ XOR } y_i \)).
8.D. (Not for submission) Consider, if you must, the language

\[
\text{Middle}(L) := \{ y \in L \mid xyz \in L \text{ for some } x, z \text{ such that } |x| = |y| = |z| \}.
\]

Prove that this language is regular.

9 (100 pts.) Fooling Sets

Prove that the following languages are not regular by providing a fooling set. You need to provide an infinite set and also prove that it is a valid fooling set for the given language.

9.A. (20 pts.) \( L = \{w w^R w \mid w \in \{0, 1\}^* \} \).
9.B. (20 pts.) \( L = \{0^i 10^j \mid i \text{ is divisible by } j \} \).
9.C. (20 pts.) \( L = \{a^i b^j \mid i, j \in \mathbb{N}, \text{ and } j = \log_2 i \} \).
9.D. (20 pts.) \( L = \{0^i 0^j \mid i, j \in \mathbb{N}, \text{ and } j = \sqrt{i} \} \).
9.E. (20 pts.) \( L = \{wcd^{\#_a(w)} \mid w \in \{a, b\}^* \} \).