Lecture 4: DFAs

Formal def'n: Product construction

Midterm 1: Sep 30, 7-9 pm
Midterm 2: TBA

Final (tentative): Dec 17 (Tue) 1:30-4:30

\[ \delta(q_0, 1) = q_1 \]

"M"

M accepts a string w if (unique) walk starting at start state and following transitions based on symbols of w ends in an accepting state.

\[ 001 \]
\[ q_0 \to q_1 \to q_2 \to q_0 \]

\[ \Sigma - alphabet \]
\[ Q - states \]
\[ s \in Q - start \]
\[ AC Q - accepting states \]
\[ \delta: Q \times \Sigma \to Q - transition function \]

Diagram:

- States: \( q_0, q_1, q_2 \)
- Alphabet: \( 0, 1 \)
- Transition function:
  - \( \delta(q_0, 0) = q_0 \)
  - \( \delta(q_0, 1) = q_1 \)
  - \( \delta(q_1, 0) = q_2 \)
  - \( \delta(q_2, 1) = q_0 \)

- Start state: \( q_0 \)
- Accepting states: \( q_2 \)

Example:

- \( 0110^* \)
  - First step: \( 0 \to q_0 \)
  - Second step: \( 1 \to q_1 \)
  - Third step: \( 1 \to q_2 \)
  - Fourth step: \( 0 \to q_0 \)
  - Fifth step: \( 1 \to q_1 \)
  - Sixth step: \( 0 \to q_2 \)
  - Seventh step: \( 0 \to q_0 \)

- Since the walk ends in an accepting state, \( 0110^* \) is accepted.
in state | in char | new state
---|---|---
$q_0$ | 1 | $q_1$
$q_0$ | 0 | $q_2$
$q_0$ | 1 | $q_3$
$q_1$ | 0 | $q_0$

DFA $M = (\Sigma, Q, q_0, F, \delta)$

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\delta(q_0, 0) = q_1
\delta(q_1, 0) = q_0
\delta(q_0, 0) = q_3
\delta(q_3, 1) = q_0
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1001 is accepted by $M = (\Sigma, q_0, F, \delta)$

$001$ is accepted by $M = (\Sigma, q_0, q_0, F, \delta)$

$011$ is not accepted by $M = (\Sigma, q_0, F, \delta)$

$101$ is accepted by $M = (\Sigma, q_0, q_0, F, \delta)$
Given $\delta$ define $\delta^* : Q \times \Sigma^* \rightarrow Q$

$\delta^*(q, e) = q$

$\delta^*(q, ax) = \delta(\delta(q, a), x)$

$\delta^*(s, 1001) = \delta(\delta(\delta(s, 1), 0), 0))$

$M$ accepts $w \in \delta^*(s, w) \in A$

$L(M)$ is language accepted by DFA $M = (\Sigma, Q, s, A, \delta)$

$L(M) = \{ w \in \Sigma^* | \delta^*(s, w) \in A \}$

$L(M) = (01+10)^*$

our running example DFA "D" in DFA $\rightarrow \delta$ is a total function defined for all pairs in $Q \times \Sigma$
\[ \delta(q, c) = \emptyset \text{ for any non-labeled trans.} \]

\[ L_{\text{odd}} = \text{odd length strings} \]

\[ L_{\text{even}} = \Sigma^* - L_{\text{odd}} = \overline{L_{\text{odd}}} \]

\[ M = (\Sigma, Q, s, A, \delta) \]
\[ M' = (\Sigma, Q, s, Q-A, \delta) \]

Langs accepted by DFAs closed under complement
(if a DFA accepts \( L \) then some other DFA accepts \( \varepsilon^* - L = \overline{L} \))

\[ L_1 = L(M \text{ odd}) \]

\[ L_2 = L(M \text{ even0}) \]

\[ L_3 = L_1 \cap L_2 \]

accept \((\text{dfa, string})\): true/false
accept \(-L_3\) (string): return accept \((L_1, \text{string})\) and accept \((L_2, \text{string})\)
\[ M = \left( Q, \Sigma, s_0, \delta_1 \right) \]
\[ M_2 = \left( Q_2, \Sigma, s_2, \delta_2 \right) \]

\[ Q = Q_1 \times Q_2 \]
\[ S = \left\{ (s_1, s_2) \right\} \]
\[ A = \left\{ (q_1, q_2) \in Q_1 \times Q_2 \mid q_1 \in A_1 \text{ and } q_2 \in A_2 \right\} \]

\[ \delta \left( (q_1, q_2), a \right) = \left( \delta_1(q_1, a), \delta_2(q_2, a) \right) \]

**Theorem:** \[ M = \left( Q, \Sigma, S, A, \delta \right) \] above as

\[ L(M) = L(M_1) \cap L(M_2) \]

**Proof**

**Lemma:** \[ \delta^* \left( (q_1, q_2), w \right) = \left( \delta_1^*(q_1, w), \delta_2^*(q_2, w) \right) \]
Proof by induction

\[ w \in L(M) \iff \exists (s_1, s_2, w) \in A \]
\[ \iff (\delta^*(s_1, w), \delta^*(s_2, w)) \in A \]
\[ \iff (s_1, w) \in A_1 \text{ and } (s_2, w) \in A_2 \]
\[ \iff w \in L_1 \text{ and } w \in L_2 \]

\[ Q_1 = \{(q_0, q_1)\} \]
\[ Q_2 = \{(q_0, q_1)\} \]

\[ Q = Q_1 \times Q_2 \]

\[ = \{(q_0, q_0), (q_0, q_1)\} \]

\[ = \{(q_0, q_0), (q_0, q_1)\} \]