**Undecidable — No algorithm**

Problems about the behavior of machines/algos.

**Halting problem:** Given code \(<M>\) and a string \(w\), does \(M\) halt given input \(w\)?

**SELF HALT:** Given \(<M>\) does \(M\) halt on \(<M>\)?

Suppose \(SH\) decides \(SELF HALT\)

\[
\text{Accept}(SH) = SELF HALT \\
\text{Reject}(SH) = \emptyset^* \setminus SELF HALT
\]

\(SH^*(w)\):

- if \(SH(w)\) accepts: hang
- else: accept

Accept \((SH^*) = \text{Reject}(SH)\)

If \(SH^*\) accepts \(<SH^*>\) \(\Rightarrow\) \(SH\) accepts \(<SH^*>\)

\(\Rightarrow SH^*\) hangs on \(<SH^*>\)

\(\Rightarrow SH\) rejects \(<SH^*>\)

\(\Rightarrow SH^*\) accepts \(<SH^*>\)

\(HALT\) is undecidable.

Suppose \(H\) decides \(HALT\)

Write \(SH(w)\):

- verify \(w\) is encoding of some \(M\)
- return \(H(w, w)\).
NeverHALT: Given $(M)$, does $M$ always halt?

Suppose $NH$ decides $NeverHALT$.

\[ H(<M>, w) : \]

- **Write the following code:**
  
  \[ M_w(x) : \]
  \[ \text{return } M(w) \]
  \[ \text{return } \neg NH(<M_w>) \]

- **Suppose $M$ halts on $w$:**
  - Then $M_w$ halts on all inputs.
  - So $NH$ rejects $<M_w>$.
  - So $H$ accepts $<M>, w$.

- **Suppose $M$ hangs on $w$:**
  - So $M_x$ hangs on all inputs.
  - So $NH$ accepts $<M_w>$.
  - So $H$ rejects $<M>, w$.
Rice's Theorem

Given \( (M) \), does \( M \) accept \( w \)?

\[
\text{Accept}(M) = \{ w \mid M \text{ accepts } w \}
\]

Let \( L \) be any set of languages such that

- There is a program \( Y \) s.t. \( \text{Accept}(Y) \in L \)
- There is a program \( N \) s.t. \( \text{Accept}(N) \notin L \)

Then deciding if \( \text{Accept}(M) \in L \) is impossible for all \( M \)

Proof (sketch):
Assume \( \emptyset \notin L \)

Suppose \( Y \) accepts language in \( L \).

Suppose \( \text{MAGIC} \) decides if \( \text{Accept}(M) \in L \)

Build

\[
H(\langle M, w \rangle)
\]

write this code:

```
WTF(x):
    call M(w)
    return Y(x)
return MAGIC(\langle WTF \rangle)
```
• Does $M$ accept $\varepsilon$?
  
  $L = \text{languages that contain } \varepsilon$
  
  $Y = \text{accept everything}$
  
  $N = \text{reject everything}$

• Does $M$ accept $\text{ILLUMINATI}$?

• Does $M$ accept only $\text{ILLUMINATI}$?

• Does $M$ accept all palindromes whose length is $2^\text{prime}$?

• Does $M$ accept either $\emptyset$ or $\Sigma^*$?

• Does $M$ accept a non-regular language?
  
  $Y = \text{accept all palindromes, nothing else}$
  
  $N = \text{accept everything}$