Prove $X$ is $NP$-hard.

Reduce known $NP$-hard problem to $X$ in poly time.

Default

- CircuitSat
- 3SAT
- Max Clique
- Max Ind Set
- Min Vertex Cover
- 3 Color
- Min Colors
- Hamiltonian Cycle/Path
- Partition/SubsetSum
- 3Partition

Binary choices

- Largest/Smallest subset
- More than 2 subsets/few subsets possible
- Ordering/path/cycle possible
- Long sequence
- Balance ferry problem!
A subset $S$ of vertices in an undirected graph $G$ is half-independent if each vertex in $S$ is adjacent to at most one other vertex in $S$. Prove that finding the size of the largest half-independent set of vertices in a given undirected graph is NP-hard.

To build $H$

- Add a new leaf to every vertex of $G$. Call new leaves $L$.

⇒ Let $S$ be any independent set in $G$.
  Then $S \cup L$ is a half-ind set in $H$.

⇒ Let $S$ be any half-ind set in $H$.

Claim: Some largest $\frac{1}{2}$-ind set in $H$ contains $L$.

Move/add marks to $L$.

⇒ new $\frac{1}{2}$ ind set $S$!

same size

bigger

Now $S' \setminus L$ is indep set in $G$.!
A subset $S$ of vertices in an undirected graph $G$ is sort-of-independent if if each vertex in $S$ is adjacent to at most 374 other vertices in $S$. Prove that finding the size of the largest sort-of-independent set of vertices in a given undirected graph is NP-hard.

Reduce from Max Ind Set

If $S$ is ind set in $G$

then $SUL$ is sort of ind set in $H$

\[
\begin{align*}
\Rightarrow & \quad \text{Largest sort of ind set } S \text{ in } H \\
\text{Move marks to leaves } \quad \Rightarrow S' \\
S' \setminus \Lambda \text{ is ind set in } G.
\end{align*}
\]
A subset $S$ of vertices in an undirected graph $G$ is almost independent if at most 374 edges in $G$ have both endpoints in $S$. Prove that finding the size of the largest almost-independent set of vertices in a given undirected graph is NP-hard.

Reduce from MaxIndepSet!

Let $S$ be the largest almost independent set in $H$.

$\Rightarrow$ Suppose some vertex in $W$ is unmarked.

Suppose some edge in $G$ is bad.

$\Leftarrow$ Unmark one end of $e$.

Mark some vertex in $W$.

New almost independent.

$|S'| = |S|$

$S' \setminus W$ is independent in $G$. 

Charon needs to ferry \( n \) recently deceased people across the river Acheron into Hades. Certain pairs of these people are sworn enemies, who cannot be together on either side of the river unless Charon is also present. (If two enemies are left alone, one will steal the obol from the other’s mouth, leaving them to wander the banks of the Acheron as a ghost for all eternity. Let’s just say this is a Very Bad Thing.) The ferry can hold at most \( k \) passengers at a time, including Charon, and only Charon can pilot the ferry.

Prove that it is NP-hard to decide whether Charon can ferry all \( n \) people across the Acheron unharmed (aside from being, you know, dead). The input for Charon’s problem consists of the integers \( k \) and \( n \) and an \( n \)-vertex graph \( G \) describing the pairs of enemies. The output is either TRUE or FALSE.

\[ k = 2V - l + 1 \]

\[ \Rightarrow \text{Suppose } S \text{ is ind set of size } l \text{ in } G \]

\[
\begin{align*}
\text{(1)} & \quad S \xrightarrow{(V \cup V') \setminus S} \emptyset \\
\text{(2)} & \quad S \xrightarrow{(V \cup V') \setminus (S \cup S')} S' \\
\text{(3)} & \quad \emptyset \xrightarrow{(V \cup V') \setminus S'} S'
\end{align*}
\]